B5.4 consultation session 1

Email questions:

2017 Q2 crests ✓

2018 Q1(c)(iii) ✓

2018 Q2(c), A value ✓

2013 Q1, p'ε p'ε strategy ✓

2013 Q2 (b)(ii) answer ✓ ; (c)(ii) hints

2016 Q3, chart diagram

2012 Q2(c)(iii), $\theta =$

2012 Q3(b)(ii), strategy

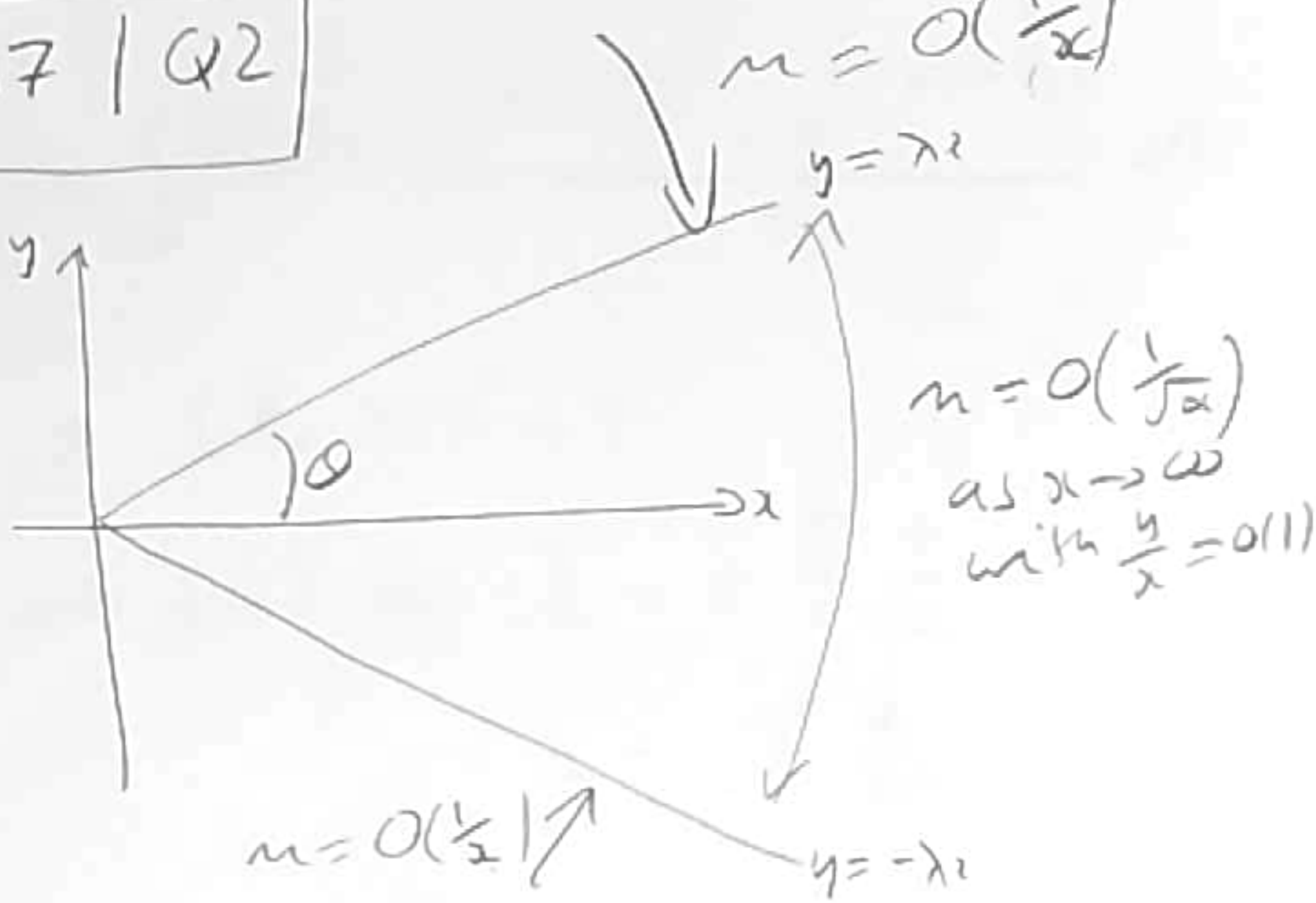
2018 Q1 (c)(iii)

with put everything on course material website

To be continued next Monday!

2017 | Q2

(c)



$\lambda = \frac{1}{2\sqrt{2}}$, $\tan\theta = \frac{1}{2\sqrt{2}}$, $\sin\theta = \frac{1}{3}$

see sheet 3, Q1 for details.

(d) $\phi = A e^{i(\underline{k}\cdot\underline{x} - \omega t)}$

$\underline{c}_p = \frac{\omega \underline{k}}{|\underline{k}|^2}$ phase velocity

$\underline{k}$ = wavenumber vector

wavelength $\frac{2\pi}{|\underline{k}|}$

$$\eta = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{\eta}_0(l)\,\frac{e^{i(kx+ly)} + e^{i(-kx+ly)}}{2}\,dl$$

$$\underline{k} = (k, l)$$

In wake $s = 2 \Rightarrow$ $l = \mp \frac{\sqrt{2}}{2}\frac{g}{U^2}$

$k = \sqrt{\frac{3}{2}}\frac{g}{U^2}$

$\Rightarrow$ angle is $\tan^{-1}\left(\frac{1}{3}\right)$

e wavelength is $\frac{2\pi}{|\underline{k}|}$

Fill in details!

2018 Q1(c)(iii)

$\phi = \phi(r,t) \Rightarrow \quad (r\phi)_{tt} = c_0^2 (r\phi)_{rr}$ for $r<a$

with $\phi_r = -i\omega a\varepsilon e^{-i\omega t}$ on $r=a$

and ϕ bdd as $r \to 0$, after linearization.

[Normal modes: $r\phi = \sin\left(\frac{\omega r}{c_0}\right) e^{-i\omega t}$
where $\omega = \omega_n$, $n \in \mathbb{N} \setminus \{0\}$]

Let $r\phi = f(r) e^{-i\omega t}$

$$\Rightarrow \quad f'' + \frac{\omega^2}{c_0} f = 0 \text{ for } r<a$$

with $\frac{d}{dr}\left(\frac{f(r)}{r}\right)\Big|_{r=a} = -i\omega a\varepsilon \quad (+)$

and $f(0) = 0$

$$\Rightarrow \quad f(r) = A\sin\left(\frac{\omega r}{c_0}\right)$$

where A s.t. $(+)$ is true

Works for $\omega \neq \omega_n \ \forall n$

What if $\exists m$ s.t. $\omega = \omega_m = \frac{c_0 l_m}{a}$?

Let $r\phi(r,t) = (f(r) + t\, g(r))\, e^{-i\omega_m t}$,

a secular soln, then <u>can</u> solve for f and g!

$$\Rightarrow \phi = \left[\frac{F + tE}{r} \sin\left(\frac{\omega_m r}{a}\right) + \frac{iE}{c_0} \cos\left(\frac{\omega r}{c_0}\right) \right] e^{-i\omega_m t}$$

where E & F are determined by BCs.

(Can add on normal modes!)

NB: $$f'' + \frac{\omega^2}{c_0^2} f = -\frac{2i\omega}{c_0^2} g$$

$$g'' + \frac{\omega^2}{c_0^2} g = 0$$

by equating powers of t in PDE.

$\boxed{\text{2018 Q2(c)}}$

$$A = -\frac{W\sin(k_* a)}{\pi k_*}\left(\frac{2\pi}{\frac{15}{4}\sigma k_*^{1/2}}\right)^{\frac{1}{2}}$$

$$k_* = \left(\frac{2V}{5\sigma}\right)^{2/3}$$

$\boxed{\text{2013 Q1}}$

Q1(a) $\Rightarrow$ $p' = c_0^2 \rho'$, $p' = -\rho_0 \phi_t$

where $\phi_{tt} = c_0^2 \nabla^2 \phi$, $c_0^2 = \frac{dp}{d\rho}(\rho_0)$.

as in §2.1 of online notes.

(b) $\phi = \phi(x,t) \Rightarrow \phi_{tt} = c_0^2 \phi_{xx}$ for $x>0$

with $\phi_x = u = -i\omega a e^{-i\omega t}$ on $x=0$

and radiation condition (of only outward travelling waves as $x \to +\infty$)

Let $\phi = f(x)e^{-i\omega t}$ (vs $f(x)g(t)$)

$$\Rightarrow \phi = \underbrace{A e^{i\omega(\frac{x}{c_0} - t)}}_{\rightarrow} + \underbrace{\cancel{B} e^{-i\omega(\frac{x}{c_0}+t)}}_{0 \quad \leftarrow}$$

(c) (ii) Let $X = x_0 e^{-i\omega t}$

KBCs $\Rightarrow$ $\phi_x = -i\omega x_0 e^{-i\omega t}$ on $x = 0_-$ (1)
and $x = 0_+$ (2)

N II $\Rightarrow$ $(-i\omega)^2 x_0 e^{-i\omega t} = [p']_{0-}^{0+}$

$= [-\rho_0 \phi_t]_{0-}^{0+}$ (3)

Unknowns : R, T, x_0

Equations : (1), (2), (3)

(iii) $\omega \to 0 \Rightarrow R \to 0, T \to 1$

$\omega \to \infty \Rightarrow R \to 1, T \to \infty$

Low frequency waves penetrate walls more effectively than high frequency ones.

$\boxed{2013,\ Q2}$

(b) $(1-M^2)\phi_{xx} + \phi_{yy} = 0$ for $y>0$

$p' = -\rho_0 U f(x)$ on $y=0$

But $p' = -\rho U \phi_x \Rightarrow \phi_x = f(x)$ on $y=0$

$\nabla\phi \to \underline{0}$ as $x^2+y^2 \to \infty$

$$\Rightarrow \phi = \frac{-1}{\pi}\int_{-\infty}^{\infty} f(\xi)\tan^{-1}\left(\frac{x-\xi}{y\sqrt{M^2-1}}\right)d\xi$$

Cf. sheet 3, Q2.