B4.2 Functional Analysis II: Q2(a)(ii) – 2017

Luc Nguyen

University of Oxford

May 2020

Given:

- $X = C([0,1])$ with the standard norm $\|\cdot\|_\infty$.
- $A_n : X \to X$ defined by $(A_n x)(t) = x(t^{1+\frac{1}{n}})$.

Want: A_n converges strongly to the identity operator on X, i.e. for every $x \in X$, $\|A_n x - x\| \to 0$.

Proof

- A_n converges strongly to the identity operator on X means: for every $x \in X$, $\|A_n x - x\| \to 0$.
- Fix $x \in X$. We need to show

$$\sup_{t\in[0,1]} |x(t^{1+\frac{1}{n}}) - x(t)| \to 0 \text{ as } n \to \infty.$$

- By uniform continuity of x, it suffices to show

$$\sup_{t\in[0,1]} |t^{1+\frac{1}{n}} - t| \to 0 \text{ as } n \to \infty.$$

- Fix some small $\epsilon > 0$.
  + If $t \leq \epsilon$, then $|t^{1+\frac{1}{n}} - t| < \epsilon$ for all n.
  + If $t > \epsilon$, then $|t^{1+\frac{1}{n}} - t| \leq |t^{\frac{1}{n}} - 1| \leq 1 - \epsilon^{\frac{1}{n}} < \epsilon$ for all large n.