C5.12 Math Physiol 2012 q1

1/ (a)

$$\epsilon \dot{v} = Av(v-a)(1-v) - w$$

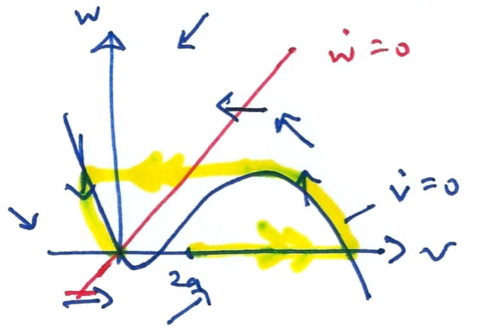
$$\dot{w} = -w + bv$$

$\epsilon \ll 1, a \ll 1$

(i) v intracellular electrical potential relative to resting state

space clamped – forced to be independent of spatial variable x

ii.

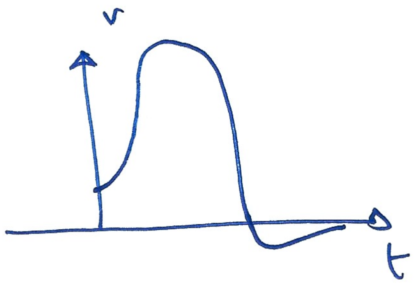


iii At large w, $\dot{v} < 0$, $\dot{w} < 0$ => trajectory arrows as shown

However, $\epsilon \ll 1$ => almost horizontal away from v nullcline

=> excursions shown (yellow)

iv



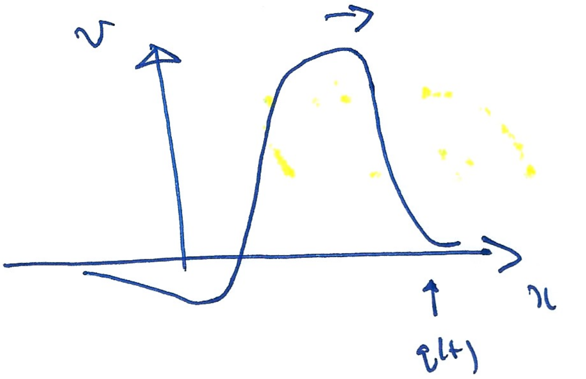
(b)

$$\varepsilon v_t = \varepsilon^2 v_{xx} + Av(v-a)(1-v) - (1+\delta \sin t)w$$

$$w_t = -w + bv$$

$v, w \to 0$ at $\pm\infty$

$\delta \ll 1$



(i) $y = x - q(t)$ $\quad \tau = t$

$$\Rightarrow \frac{\partial}{\partial x} = \frac{\partial}{\partial y}, \quad \frac{\partial}{\partial t} = \frac{\partial}{\partial \tau} - \dot{q}\frac{\partial}{\partial y}$$

$$\Rightarrow \quad w_\tau = \dot{q} w_y - w + bv$$

$$\& \quad \varepsilon(v_\tau - \dot{q} v_y) = \varepsilon^2 v_{yy} + Av(v-a)(1-v) - (1+\delta \sin \tau) w$$

(ii) ~~Now we~~ Now I suppose ~~we~~ ahead of the front, still on $y \sim O(1)$,

w and v are small, neglect ε

$$\Rightarrow \quad Av(v-a)(1-v) \simeq \cancel{Av} \simeq -aAv \approx (1+\delta \sin \tau) w$$

so $v \simeq -\dfrac{(1+\delta \sin \tau) w}{aA}$ $\quad$ ah $a \ll 1$ but also $\delta \ll 1$... or.

suggests $w_\tau - \dot{q} w_y \simeq -w - \dfrac{b(1+\delta \sin \tau)}{aA} w$

$$= -w\left[1 + \frac{b(1+\delta \sin \tau)}{aA}\right]$$

of required form with

$$f(\tau) = 1 + \frac{b(1+\delta\sin\tau)}{aA}$$

– well this is a bit loose as $a \ll 1$ but I suppose ok if $\varepsilon \ll 1$

iii) $$w_\tau - \dot q\, w_y = -f w$$

so $f > 0$ & $\dot q > 0$

can solve with characteristics

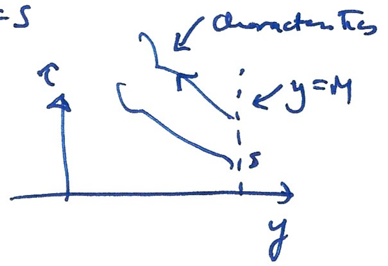
Note $\dot y = -\dot q < 0$ characteristics emerge from $y = \infty$!

Since f = constant (as $\delta \ll 1$)

~~Suppose $w = w_\infty(s)$ at $y = s > 0$ at $\tau = 0$~~ $\tau = s$, y

~~then $\dot y = -\dot q \Rightarrow y = q(s) - q(\tau)$~~

Suppose we apply $w = w_M(s)$ at $y = M$ and $\tau = s$



then $$\dot w = -f w$$
$$\dot y = -\dot q$$
$$\Rightarrow y = M - [q(\tau) - q(s)]$$
$$w \simeq w_M(s)\, e^{-f(\tau - s)}$$

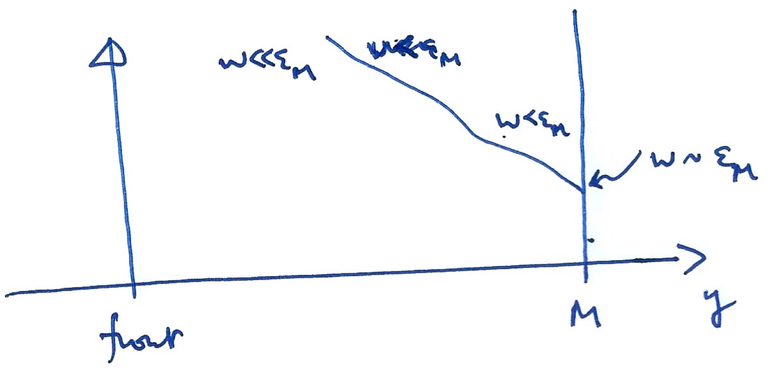
Since $\delta \ll 1$ we have approximately $q = c\tau$ (convenient assumption)

so $$\tau - s \simeq \frac{M-y}{c}$$

$$\Rightarrow w \simeq w_M\left[\tau - \frac{(M-y)}{c}\right] \exp\left[\frac{y-M}{c}\right]$$

& in order for this to $\to 0$ at ∞ we need $w_M \to 0$ rapidly $\Rightarrow w \equiv 0$.

This is not very well put. Basically w decays exponentially along the characteristics as y ~~does~~ decreases, so if $w \to 0$ as $y \to \infty$, it enforces $w = 0$ ahead of the front.



If $M \gg 1$, $w \sim \varepsilon_M \ll 1$ for $y \sim 1$ for sufficiently large τ.

It's a bit awkward.

(iv) well, if we take $w = 0$ ahead of the front

and now in the front $y = \varepsilon Y$

at leading order $\Rightarrow$ $-\dot{q}\, v_Y = v_{YY} + Av(v-a)(1-v)$ as $w = 0$

$v \to 0 \quad Y \to \infty$
$v \to 1 \quad Y \to -\infty$

& using the limit

$$v'' + \dot{q} v' - Av(v-a)(v-1)$$

$U_0 = 0,\ U_1 = a,\ U_2 = 1$ $\quad$ $v \to U_0 = 0 \quad Y \to \infty$, $\quad v \to U_2 = 1 \quad Y \to -\infty$

$$\Rightarrow c = \sqrt{\frac{A}{2}}\left(U_0 + U_2 - 2U_1\right)$$
$$= \sqrt{\frac{A}{2}}(1-2a) > 0.$$

Rather weird question