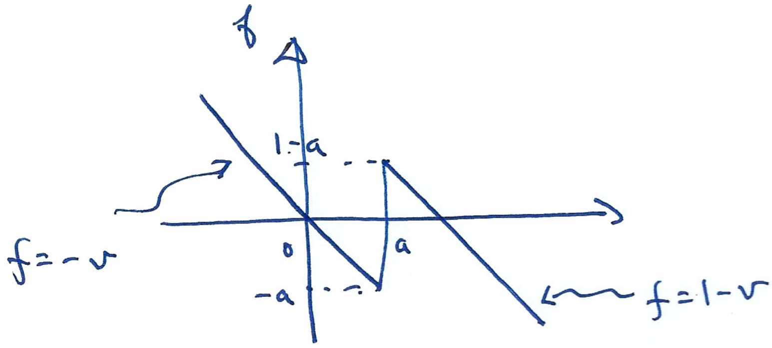
C5.12 Math Physiol 2015 q1 answer.

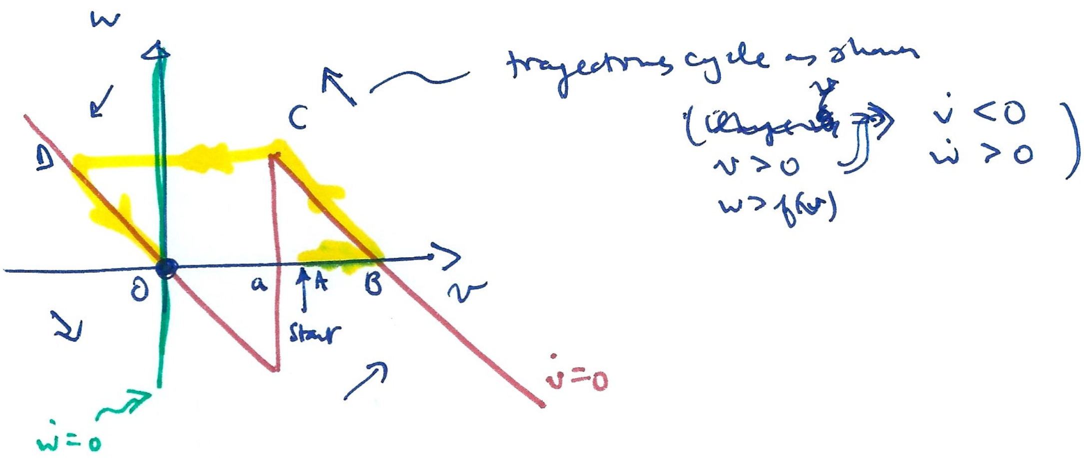
1, (a)

$$\varepsilon \dot{v} = f(v) - w$$
$$\dot{w} = v$$



(i) v electrical potential etc

w gate variable etc



(0,0) unique equilibrium

Near (0,0)
$$\begin{pmatrix}\dot{v}\\ \dot{w}\end{pmatrix} \approx \underbrace{\begin{pmatrix}-\frac{1}{\varepsilon} & -1\\ 1 & 0\end{pmatrix}}_{M}\begin{pmatrix}v\\ w\end{pmatrix} \text{ etc}$$

$\det M = 1$

$\operatorname{tr} M = -\frac{1}{\varepsilon}$

$\Rightarrow$ stable

(2

Starting at A $(v^*, 0)$ $v^* > a$ trajectory is as

shown: ABCDO : AB fast to v nullcline as $\varepsilon \ll 1$

BC slow

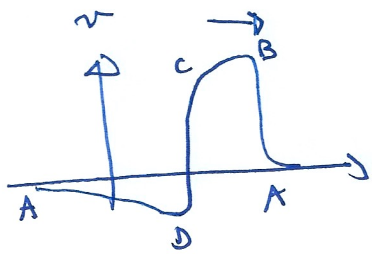
CD fast

DO slow

$\Rightarrow$ steady state is excitable

(b) $\varepsilon v_t = \varepsilon^2 v_{xx} + f(v) - w$

$w_t = v$



$y = x - ct$, $c > 0$

$v = v(y)$, $w = w(y)$

(i) $-\varepsilon c v' = \varepsilon^2 v'' + f(v) - w$

$-cw' = v$

(ii) $y = \varepsilon Y$ $\Rightarrow$ $-cv' = v'' + f(v) - w$

in front AB $w' = \varepsilon v$

$\Rightarrow w' \simeq 0 \Rightarrow w = 0$ $(w \to 0$ as $Y \to \infty)$

$\Rightarrow v'' + cv' + f(v) = 0$

bcs $v \to 0$ as $Y \to \infty$

$v \to 1$ (only other physically stable steady state) as $Y \to -\infty$

iii

we have to get from $v = 1$ at $y = -\infty$

to $v = 0$ at $y = +\infty$

for $v > a$ $\quad v'' + cv' + 1 - v = 0$

$f = 1 - v$

solutions exponentials

& $1 - v \propto e^{\lambda y}$

$$-\lambda^2 - c\lambda + 1 = 0$$

$$\lambda^2 + c\lambda - 1 = 0$$

$$\lambda = \tfrac{1}{2}\left[-c \pm \{c^2+4\}^{1/2}\right] = \lambda_\pm \qquad \lambda_+ > 0, \; \lambda_- < 0$$

we need $\lambda > 0$ so $1 - v \to 0$ as $y \to -\infty$

$\Rightarrow \quad 1 - v = e^{\lambda_+ y}$ $\quad$ (coeff $= 1$ wlog as just fixes origin)

till $v = a$ at $y = \frac{1}{\lambda_+} \ln(1-a) = y_a$

& $v' = -\lambda_+ e^{\lambda_+ y_a} = -\lambda_+(1-a)$

for $v < a$ $\quad v'' + cv' - v = 0$ $\quad$ solutions $e^{\lambda_\pm y}$ as before

we need λ_- so $v \to 0$ as $y \to \infty$

thus & require $v = a e^{\lambda_-(y - y_a)}$

& v' cts at y_a so $\lambda_- a = -\lambda_+(1-a)$

or $\tfrac{1}{2}\left[c + (c^2+4)^{1/2}\right] a = \tfrac{1}{2}\left[-c + (c^2+4)^{1/2}\right](1-a)$

4

So $ca + c(1-a) = -(c^2+4)^{1/2} a + (c^2+4)^{1/2}(1-a)$

~~i.e. $c(2-a) = (c^2+4)^{1/2} a$~~

$\Rightarrow c = (c^2+4)^{1/2}(1-2a)$ requires $a < \frac{1}{2}$ for $c > 0$

$\Rightarrow c^2 = (c^2+4)(1-2a)^2$

~~$= c^2(1-2a) + 4(1-2a)$~~

~~$\Rightarrow c^2 = 2(1-2a)$~~

$c^2\left[1-(1-2a)^2\right] = 4(1-2a)^2$

$\parallel$

$c^2\left[4a - 4a^2\right]$

So $c = \dfrac{1-2a}{[a(1-a)]^{1/2}}$

(iv) well come on you already told us c was +ve

anyway what he wants is

$v'' + cv' + f(v) = 0$

$\frac{1}{2}v'^2\Big|_{-\infty}^{\infty} + c\int_{-\infty}^{\infty} v'^2\,dy + \int_1^0 f(v)\,dv = 0$

i.e. $c = \dfrac{\int_0^1 f(v)\,dv}{\int_{-\infty}^{\infty} v'^2\,dy} \gtrless 0$ if $\int_0^1 f(v)\,dv \gtrless 0$

evidently if $a \lessgtr \frac{1}{2}$

(this sign is wrong)