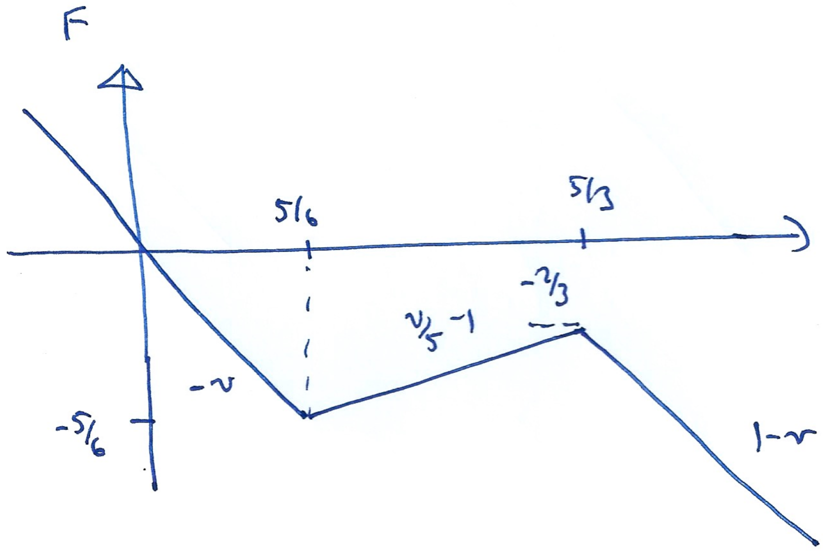
# CS12 Math Physiol 2016 q1

(a) $\varepsilon \dot{v} = I + f(v) - w$

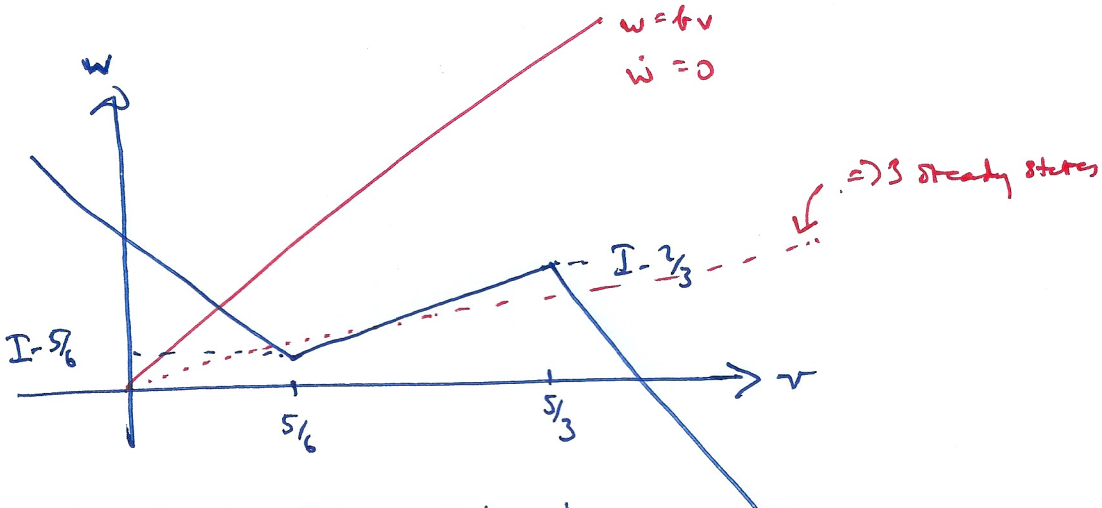
$\dot{w} = -w + b v$ $\qquad I > 5/6$




; v intracellular electric potential

w gate variable for K channel conductance

I applied current




The dotted line indicates 3 steady states if

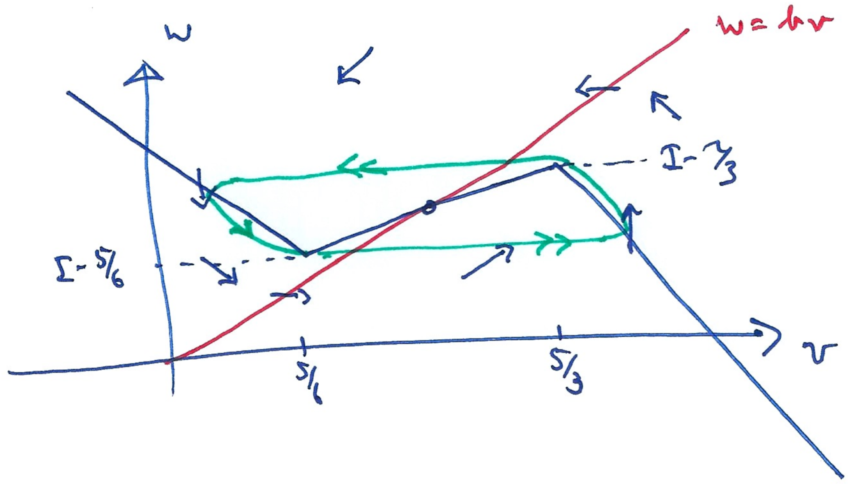
$b < \frac{1}{5}$ and

$\frac{5}{6} b > I - \frac{5}{6}$ and $\frac{5}{3} b < I - \frac{2}{3}$

viz $\quad \frac{5b+2}{3} < I < \frac{5b+5}{6}$ (which requires $b < \frac{1}{5}$)

(b) 1 steady state $\frac{5}{6} < v^* < \frac{5}{3}$ requires

(i)




Steady exists for

$$\frac{5}{6}b < I - \frac{5}{6} \quad \& \quad \frac{5}{3}b > I - \frac{7}{3}$$

i.e. $\frac{5}{6}(b+1) < I < \frac{5b+2}{3}$ (Note $b > \frac{1}{3}$)


At large w $\quad \dot{v} < 0 \quad \dot{w} < 0 \quad$ so trajectory directions as for arrows above in diagram

$\Rightarrow$ fixed point is node or spiral.

$\varepsilon \ll 1 \Rightarrow$ relaxation oscillations as indicated in green above

ii $\Rightarrow$ steady state is unstable

(but to do it formally, unstable if $\mathrm{tr}\, M = T > 0$

(det $M > 0$ as not a saddle)

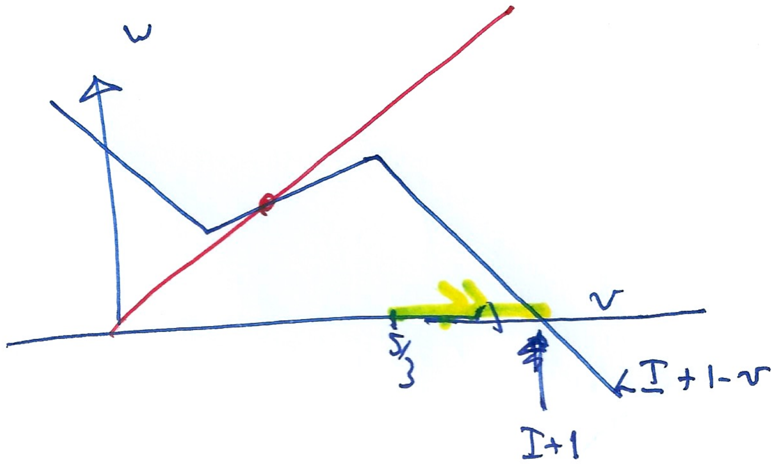
$M = \begin{pmatrix} F'/\varepsilon & -1/\varepsilon \\ b & -1 \end{pmatrix} \quad \mathrm{tr}\, M = \frac{F'}{\varepsilon} - 1 > 0$ as $\varepsilon \ll 1$ & $F' > 0$.

Depends on $\varepsilon$ & $b$ ? This is a bit vague as we are told $\varepsilon \ll 1$.

$T = \frac{1}{5\varepsilon} - 1$ so unstable for $\varepsilon < 0.2$.

Only depends on $b$ if steady state not on middle branch.

iii




Start at $(5/3, 0)$ in yellow

$t = \varepsilon\tau$ $\quad v' = I + f(v) - w$

$w' = \varepsilon[-w + b]$

$\Rightarrow w \simeq 0, \quad v' \simeq I + f(v) = I + 1 - v$

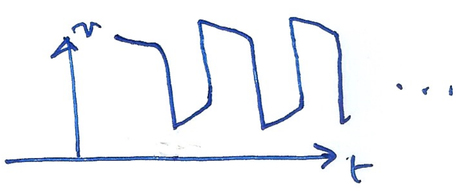
$v \to I+1$

iv Already done in (i): v relaxes to v nullcline rapidly

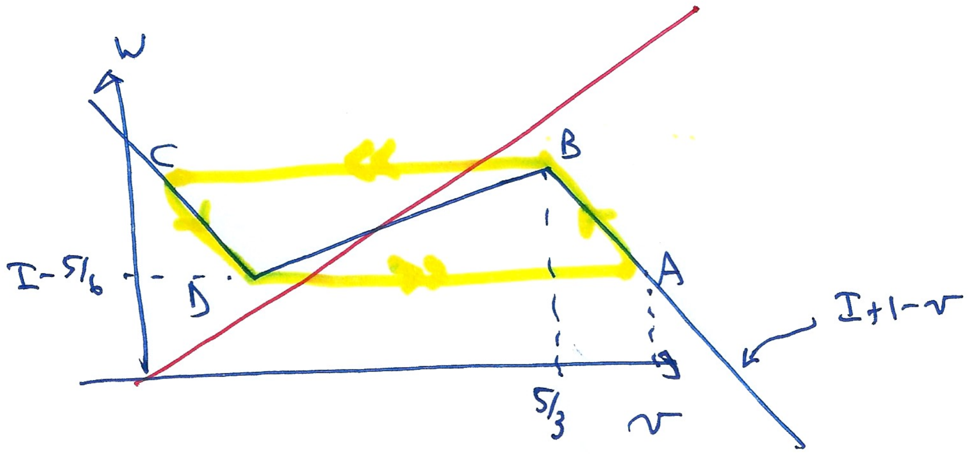
w trundles slowly on v nullcline towards red

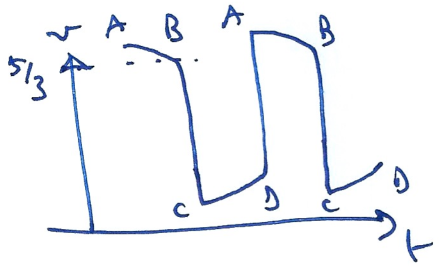
w nullcline but can't stay on v nullcline so hops rapidly

to other branch



T = length of time $v > 5/3$ in oscillation :




$$T = \int_A^B dt = \int_A^B \frac{dw}{\dot{w}} = \int_A^B \frac{dw}{-w + bv}$$

and $w = I + 1 - v$

$$\text{so } T = \int_A^B \frac{-dv}{bv + v - (I+1)}$$

At A $I + 1 - v = I - 5/6 \Rightarrow v = \frac{11}{6}$

At B $v = 5/3$

$$\Rightarrow T = \int_{11/6}^{5/3} \frac{-dv}{(b+1)v - (I+1)}$$

$$= \frac{1}{b+1} \ln\Big[(b+1)v - (I+1)\Big]_{5/3}^{11/6}$$

$$= \frac{1}{b+1} \ln\left[\frac{\frac{11}{6}(b+1) - I - 1}{\frac{5}{3}(b+1) - I - 1}\right] = \frac{1}{b+1} \ln\left[\frac{11b + 5 - 6I}{2(5b + 2 - 3I)}\right]$$