C6.2/B2. Continuous Optimization

Resources

References

[1] A. R. Conn, N. I. M. Gould and Ph. L. Toint, *Trust-Region Methods*, SIAM 2000.

[2] J. Dennis and R. Schnabel, *Numerical Methods for Unconstrained Optimization and Nonlinear equations*, (republished by) SIAM, 1996.

[3] R. Fletcher, *Practical Methods of Optimization*, 2nd edition Wiley, 1987 (republished in paperback in 2000).

[4] P. Gill, W. Murray and M. H. Wright, *Practical Optimization*, Academic Press, 1981.

[5] N. I. M. Gould, *An Introduction to Algorithms for Continuous Optimization*, 2006. Available for download at `http://www.numerical.rl.ac.uk/nimg/course/lectures/paper/paper.pdf`.

[6] J. Nocedal and S. J. Wright, *Numerical Optimization*, Springer Verlag, 1999 (1st edition) or 2006 (2nd edition). All citations in the lecture notes apply to either edition, unless otherwise stated.

Comments on the bibliography

For a comprehensive, yet highly accessible, introduction to numerical methods for continuous (unconstrained and constrained) optimization problems, see [6] - most recommended (but not required) for this course ! Reference [5] is also a very good, but more succinct introduction to this topic, with particular emphasis on nonconvex problems and with a well-structured bibliography of fundamental optimization articles. The monograph [1] is the most comprehensive reference book on trust-region methods to date. The remaining books in the bibliography are classics of the nonlinear (constrained and unconstrained) optimization literature.

Online and software resources

For an index and a guide to existing public and commercial software for solving (constrained and unconstrained) optimization problems, see

`http://neos-guide.org/Optimization-Guide`

and follow the links to *Optimization Tree* for example. Other useful links related to optimization may be found at the same webpage (links to test problems, to the NEOS Server which solves user-sent optimization problems over the internet, to online repositories of optimization articles, etc.).

For general nonconvex, smooth constrained and unconstrained problems the following software packages are of high quality/reliable: KNITRO, IPOPT, GALAHAD, etc. MATLAB's optimization toolbox (available on departmental computers) contains built-in optimization solvers for various problem classes - be careful which subroutine you choose ! COIN-OR is a public software repository that you may find useful in the future.

An important aspect of optimization software is the *interface* that allows the user to input the problem to the solver; interfaces, and hence acceptable input formats, vary between solvers. Presently, usually besides file-input in the language the solver is written in, much software allows MATLAB input files or/and AMPL files (AMPL is a modelling language specifically designed for expressing optimization problems; see `www.ampl.com`), etc.

Optimization beyond this course

We briefly discuss generic features of the optimization models that we address in the course by placing them in the broader context of the field of optimization (with some literature pointers for some of the material that is beyond the scope of this course). The underlined classes of problems are the ones we address in the course.

A. Smooth versus nonsmooth optimization One of the main assumptions in our course will be that the objective and the constraints are sufficiently smooth functions. When differentiability requirements are absent (but the functions remain continuous), different analytic tools than the ones to be presented in this course have to be used for identifying a solution of the problems (*subgradient*, etc.). The difficulties spring from the unpredictibility of the behaviour of say the objective function near one of its points of nonsmoothness. See J. B. Hiriart-Urruty and C. Lemarechal, *Convex Analysis and Minimization Algorithms*, Springer 1991, and F. H. Clarke, *Optimization and Nonsmooth Analysis*, SIAM, 1990, for fundamental investigations of theoretical issues connected to this class of problems. When even continuity of the objective and/or constraints is lacking, then it is essential to know something about the structure of these functions or nature of discontinuities in order to have any hope of solving the problem.

B. Continuous versus discrete optimization In some optimization problems, the variables make sense only if they are integers (for example, the company in Example 2 may be a car manufacturer). *Discrete optimization* addresses these problems. Very often, a discrete optimization algorithm would solve a sequence of continuous problems (see *branch-and-bound* methods for *integer linear programming* that use linear programming *relaxations* of the integer programming problem, or see approximation algorithms using *semidefinite programming relaxations* for the *max-cut combinatorial problem* (Goemans & Williamson, 1995)). An active research area at present is *mixed integer nonlinear programming*, where the objective and/or constraints are nonlinear functions, and where there are also integrality requirements on at least some components of x. The literature for discrete (linear) optimization algorithms is vast (see, for example, A. Schrijver, *Theory of linear and integer programming*, Wiley, 1986 , and L. A. Wolsey, *Integer Programming*, Wiley, 1998).

C. Deterministic versus stochastic optimization It may happen that the model data for an optimization problem is not fully known at the time of the setting and solving of the model. For example, many financial planning and economic models share this feature, as they depend on future demand, prices and interest rates. Based on statistical estimates of the unknown parameters of the model, different *scenarios* are constructed and endowed with a certain probability. *Stochastic optimization* addresses solving these scenarios, which are deterministic problems and may be solved by methods that we study in this course, to obtain the optimal value of the expected performance of the model. See J. R. Birge and F. Louveaux, *Introduction to Stochastic Programming*, 1997.

D. Other classes of optimization problems Some more general optimization problems may include multiple objectives (*multiobjective optimization*) or infinitely many constraints (*PDE-constrained optimization*, etc., etc.), and they are beyond the scope of this course. As an example of a multiobjective optimization problem, consider car design, where an engineer may wish to maximize crash resistance for safety purposes and minimize weight for fuel consumption. New optimality concepts need to be employed for this class of problems (Pareto optimality, etc.), as $\mathbb{R}^n$ is only a partially ordered set (not any two points can be compared). See, for example, K. Miettinen, *Nonlinear Multiobjective Optimization*, Kluwer, 1999. For PDE-constrained problems, the techniques in this course are very relevant, as each problem is discretized into (many) finitely-constrained problems to be solved. The latter pose a challenge as their dimensions are (very) large. For more on the very active research area of PDE-constrained optimization, see the work and website of Omar Ghattas, Ekkehard W. Sachs, and the references therein. See also, L. Biegler, O. Ghattas, M. Heinkenschloss and B.v. Bloemenn Waanders, editors, *PDE-Constrained Optimization*, Springer, 2003, for some of the latest developments.