## Geometric Group Theory

## Problem Sheet 0

1. Show that a subgroup of index 2 is normal.

**2.** Let A, B be finite index subgroups of G. Show that  $A \cap B$  is a finite index subgroup of G.

**3.** Let G be a finitely generated group and let H be a subgroup of G of finite index. Show that H is finitely generated.

**4.** Show that if G is a finitely generated group such that every (non-trivial) element of G has order 2 then G is finite.

**5.** Let *H* be a finite index subgroup of *G*. Show that there is a normal finite index subgroup *N* of *G*, such that  $N \subset H$ .

**6.** Let G be a finitely generated group. Show that G has finitely many subgroups of index n. (*hint:* use the previous exercise).