Exercise sheet 2*

(for the class in W5)

Exercise 1.1. *Let R be a ring and $\mathfrak{p}$ a prime ideal of R. Show that there is a natural isomorphism*

$$\varinjlim_{r\in R;r\not\in\mathfrak{p}} R_r \simeq R_{\mathfrak{p}}$$

Here the arrows in the inductive system are defined as follows. If r' is a multiple of r then the arrow is the natural map $R_r \to R_{r'}$. Otherwise there is no arrow. Use this to show that Prop. 2.5 (b) implies Prop. 2.5 (c).

Exercise 1.2. *Show that if a morphism is affine (resp. quasi-compact) with respect to a certain open affine covering then it is affine (resp. quasi-compact) with respect to any open affine covering.*

Exercise 1.3. *Let $\phi : R \to T$ be a morphism of rings. Let N be an R-module and M a T-module. Show that there is a functorial isomorphism*

$$\mathrm{Mor}_R(N, M) \simeq \mathrm{Mor}_T(N \otimes_R T, M)$$

where in the expression $\mathrm{Mor}_R(N, M)$, M is viewed as an R-module via ϕ. Use this to give a quick proof of the fact that the functor $N \mapsto N \otimes_R T$ from R-modules to T-modules is a right exact functor.

Exercise 1.4. *Let X be a quasi-compact scheme. Prove that X has a closed point.*

Exercise 1.5 (optional)**.** *Prove Lemma 2.17 in the notes.*

*For the course C2.6 Introduction to Schemes, Oxford, Hilary Term 2019