Exercise sheet 3*

(for the class in W8)

Exercise 1.1. *Let $r \geqslant 2$. Prove that $\mathbb{A}^r_{\mathbb{C}} \backslash \{0\}$ is not affine.*

Exercise 1.2 (noetherian induction for schemes)**.** *Let T be a noetherian scheme. Let $P(\bullet)$ be a property of closed subschemes of T. Suppose that P(empty scheme) holds and that for all closed subschemes C of T, the statement*

$$\text{if } P(C') \text{ holds for all closed subschemes } C' \overset{\neq}{\hookrightarrow} C \text{ then } P(C) \text{ holds}$$

is verified. Then $P(T)$ holds.

Exercise 1.3. *Let X be an integral scheme and let $\phi : F \to G$ be a morphism of coherent locally free sheaves on X. Let $U \subseteq X$ be an affine open subscheme and suppose that $\phi(U) : F(U) \to G(U)$ is an injective map. Prove that ϕ is a monomorphism.*

Exercise 1.4. *Let R, J be rings and let $(f, f^\#) : \operatorname{Spec} R \to \operatorname{Spec} J$ be a closed immersion. Show that the corresponding map of rings $\phi : J \to R$ is surjective. Let $I := \ker \phi$. Show that the quasi-coherent sheaf of ideals associated with $(f, f^\#)$ is isomorphic to $\widetilde{I}$. Show that the image of f is $V(I)$.*[1] *Show that the composition of two closed immersions is a closed immersion.*

Exercise 1.5 (optional)**.** *Let X be a noetherian scheme and let X_{red} be the reduced closed subscheme of X associated with the closed subset X of X. Show that X_{red} is affine if and only if X is affine.*

*For the course C2.6 Introduction to Schemes, Oxford, Hilary Term 2019

[1] for $\widetilde{\bullet}$ and $V(\bullet)$, see sections 2.1 and 2.3 in the notes.