ASSIGNMENT 4 SOLUTIONS

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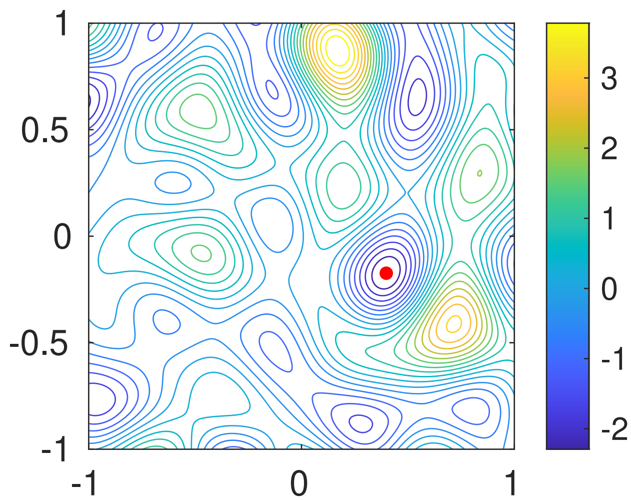
1. Chebfun

From the picture, I guessed the minimum would be around -2 and located at around $(0.4, -0.2)$. `min2` confirms these estimates, though the minimum is a bit lower than I guessed:

```
rng(1), f = randnfun2(.5);
contour(f,20), axis equal, colorbar, hold on
[val,pos] = min2(f)
hold on, plot(pos(1),pos(2),'.r','markersize',15), hold off

val = -2.610968758156774
pos = 0.398256866771717   -0.173741101821436
```



2. BFO

I found that to get `bfo` to work, I had to have my function in an external file. There must be a better way! — but anyway, what I did was to store the chebfun2 in a file called f_file:

```
save f_file f
```

My function for BFO then loads `f_file` every time it runs, which is pretty ugly. (Another method might involve passing parameters to the objective function.)

```
type fBFO

function fBFO = fBFO(x)
load foo, fBFO = f(x(1),x(2));
```

Anyway, let's run `bfo` 200 times from random starting points like this and print out whenever we encounter the minimum so far:

```
tic, minfX = 999;
for j = 1:200
  [X,fX] = bfo(@fBFO,2*rand(2,1)-1,'reset-random-seed','no-reset', ...
               'verbosity','silent','xlower',[-1 -1],'xupper',[1 1]);
  if fX < minfX + 1e-6
```

```
    fprintf('%3d (%9.6f,%9.6f) %12.8f \n',j,X(1),X(2),fX)
    minfX = fX;
  end
end
toc
```

```
  1 (-0.635860,-0.400090)  -0.76458183
  2 (-0.967926,-0.768877)  -1.89585847
  3 ( 0.544132, 0.648040)  -2.10730111
  4 (-1.000000, 0.636131)  -2.34040243
  9 (-1.000000, 0.636129)  -2.34040243
 12 (-1.000000, 0.636137)  -2.34040242
 17 (-1.000000, 0.636114)  -2.34040244
 19 (-1.000000, 0.636136)  -2.34040242
 31 (-1.000000, 0.636121)  -2.34040244
 32 (-1.000000, 0.636105)  -2.34040243
 35 ( 0.398248,-0.173751)  -2.61096875
 38 ( 0.398262,-0.173748)  -2.61096875
 49 ( 0.398267,-0.173732)  -2.61096875
 62 ( 0.398269,-0.173748)  -2.61096874
 64 ( 0.398244,-0.173737)  -2.61096874
 83 ( 0.398273,-0.173755)  -2.61096872
108 ( 0.398266,-0.173714)  -2.61096871
166 ( 0.398271,-0.173723)  -2.61096873
177 ( 0.398253,-0.173738)  -2.61096876
190 ( 0.398256,-0.173749)  -2.61096875
192 ( 0.398267,-0.173740)  -2.61096875
Elapsed time is 35.914084 seconds.
```

This is pretty good evidence of the same global minimum we found in problem 1, though of course not definitive. One could explore other options of BFO to do a more systematic search.

3. A 3D example

Here's a Rosenbrock-type function in 3D. The minimum is the value 0 at $x(1) = x(2) = x(3) = 1$.

```
ff = @(x,y,z) (x-1).^2 + 100*(y-x.^2).^2 + 100*(z-y.^2).^2;
```

With Chebfun3 we find it like this:

```
tic, f = chebfun3(ff,[-2 2 -2 2 -2 2]); [val,pos] = min3(f), toc

val = 1.762145984685048e-11
pos = 1.000004097968050   1.000008239198563   1.000016526955550
Elapsed time is 0.427421 seconds.
```

With BFO things work fine too:

```
type fBFO2
rng(1), tic
[X,fX] = bfo(@fBFO2,randn(3,1),'verbosity','silent'), toc

function fBFO2 = fBFO2(x)
fBFO2 = (x(1)-1).^2 + 100*(x(2)-x(1).^2).^2 + 100*(x(3)-x(2).^2).^2;

X =
   0.995096576184185
   0.990216392767456
   0.980509789516367
fX =
     2.407865470648813e-05
Elapsed time is 0.063166 seconds.
```