Scientific Computing for DPhil Students I
Assignment 1

Due at lecture at 10:00 on Tuesday, 22 October 2019.
This is the first of four assignments this term.
It will count for only 5% of the overall mark for the term.

In this as in all assignments in this course, the answers you turn in should be attractive and brief but complete, and should include program listings and plots where appropriate, as well as appropriate discussion. It is ok to discuss problems in a general way with other students, but your programs and your writeups must be your own.

1. Consider the 3×3 matrix

$$A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & 1 \\ 0 & 1 & 3 \end{pmatrix}.$$

Use Matlab to compute A^2, A^{-1}, and $\det(A)$. Also compute the eigenvalues of A and verify that their product equals $\det(A)$.

2. Compute the exponential e^A of the same matrix A by two methods: (a) using Matlab's `expm` command, and (b) by writing a `for` loop that sums the Taylor series $e^A = I + A + A^2/2! + \cdots$ to sufficiently many terms. How closely do the two results agree?

3. Write a program that creates the following anonymous functions to generate matrices for a given positive integer N:

```
D = @(N) sparse(toeplitz([2 -1 zeros(1,N-2)]));
I = @(N) speye(N);
A = @(N) kron(I(N),kron(I(N),D(N))) + kron(I(N),kron(D(N),I(N))) ...
               + kron(D(N),kron(I(N),I(N)));
```

What is the dimension of $A(N)$ as a function of N? List $A(2)$ explicitly. Draw a spy plot of $A(4)$. Now make another anonymous function `b = @(N) ...` to create a column vector with the same number of rows as $A(N)$, with all entries equal to 1. Apply these functions and Matlab's backslash operator to solve $Ax = b$ for $N = 10, 20, 30, \ldots, N_{\max}$ with $N_{\max} = 70$ or 80 (whatever works ok on your machine). Make a table of the times required for these computations, and also a loglog plot. As $N \to \infty$, if these were dense matrices and your machine had infinite storage and no communication limitations, with what power of N would you expect the time to scale? But this is far from a dense matrix. In fact, what power of N do the data suggest?