# Scientific Computing for DPhil Students I
# Assignment 2

*Due at lecture at 10:00 on Tues. 5 Nov. 2019. This is the second of four assignments this term.*

*Here is our policy on collaboration. By all means ask friends for help on general questions like "How do I compute a condition number in MATLAB?" or "What's a good reference on conjugate gradients?" Do not, however, discuss specifics of these four problems.*

*Make your MATLAB codes clean, elegant, and short.*

Given a dimension $n$, let $A$ be the $n \times n$ matrix generated by the commands

```
rng(1), A = 0.1*sprandsym(n,10/n) + speye(n);
```

Be sure that the `rng(1)` command is executed before each computation of $A$, so that we are all working with the same random numbers.

1. *Condition numbers by direct method.* Compute condition numbers $\kappa(A)$ for $n = 100, 200, 400, \ldots$ using `eig(A(n))`, stopping when the computation takes more than a few seconds.

2. *Condition numbers by iterative method.* Now do the same again with `eigs(A(n),'largestreal')` and `eigs(A(n),'smallestreal')`, which make use of the implicitly restarted Lanczos algorithm. How high can you go now before things get slow?

3. *System of equations by direct method.* Taking $b$ to be the vector of all ones, solve $Ax = b$ for $n = 1000, 2000, 4000, \ldots$ for $x$ using the backslash operator in MATLAB, stopping when the computation takes more than, say, 10 seconds. Plot and discuss the timing as $n \to \infty$. Estimate how long it would take for $n = 10^6$ if this behaviour continued for large $n$.

4. *System of equations by iterative method.* With $n = 10^6$, use CG to solve $Ax = b$ to ten digits of accuracy, and in particular, report the first the entry $x_{1000}$ of the solution. Plot the convergence as a function of CG iteration number and quantify how closely this matches what you would expect based on the theory of convergence of CG.

5. *A modified system of equations.* Now take $B$ to be the same matrix as $A$, except with `speye(n)` replaced by `diag(sparse(1:n))`. With $n = 10^6$ and the same right-hand side as before, find a reasonably speedy way to compute the solution of $Bx = b$ to 10 digits of accuracy and report the entry $x_{1000}$ of the solution.