Scientific Computing for DPhil Students II Assignment 4

Due at the Andrew Wiles Building reception at 10:00 on Tuesday morning of Week 8, 10 March 2020. If you prefer, you may email a pdf file to trefethen@maths.ox.ac.uk. No papers will be accepted after this hour. This is the last of four assignments this term.

Go to courses.maths.ox.ac.uk/node/45433 and review the lecture notes and demonstration M-files for this course. Then compute the following six numbers, each to at least three digits of relative accuracy (e.g. 0.665 or 23.2 or 0.00277). For each problem, also make a plot of the solution.

Use whatever methods and tools you like; they most certainly do not have to be optimal. But however you get the number, explain what you have done plainly, including crucial code segments as necessary and making it clear what evidence you have for three digits of accuracy.

- 1. What's the period of the interesting solutions of $0.05u'' (1 u^4)u' + u = 0$?
- 2. Let u_1 and u_2 be the solutions to

$$\varepsilon u'' + xu' + 2u = 0,$$
 $u(-1) = 2, u(1) = 1$

with $\varepsilon = 0.50$ and 0.51, respectively. What is $||u_1 - u_2||_{\infty}$?

- 3. What is the smallest eigenvalue λ of $-u_{xx} + x^4 u = \lambda u$ with zero boundary conditions on the interval [-5, 5]?
- 4. At what time $t = t_c$ does the solution u to the equation

$$u_t = 0.015u_{xx} + u - u^3$$
, $u(-1, t) = u(1, t) = -1$, $u(x, 0) = 1 - 2x^2$

become negative?

5. The solution to the initial boundary-value problem

$$u_t = u_{xx} + u_x + e^u$$
, $u(-1,t) = u(1,t) = 0$, $u(x,0) = 0$

blows up to ∞ at a finite time t_c . What is t_c ?

6. A conducting cube is in thermal equilibrium with temperature u = 1 on face A and u = 0 on the other five faces. What is the temperature at the point midway between the center of the cube and the center of face A?