Advanced Supersymmetry: Problem sheet 2

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Due by Friday, week 2 (March 5), 4pm.

[*You may decide to answer only one of the two questions below.*]

1. $\mathcal{N}=2$ Super-Yang mills action: extended supersymmetry

The $\mathcal{N}=2$ SYM Lagrangian reads:

$$\begin{aligned}\mathscr{L}_{\mathcal{N}=2\,\mathrm{SYM}} &= \frac{1}{g^2}\,\mathrm{tr}\Big(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - D_\mu\bar\phi D^\mu\phi - i\bar\lambda_I\bar\sigma^\mu D_\mu\lambda^I + \frac{1}{2}D^2 + \bar F F \\ &\qquad - \bar\phi[D,\phi] - \frac{i}{\sqrt{2}}\bar\phi\epsilon_{IJ}[\lambda^I,\lambda^J] + \frac{i}{\sqrt{2}}\epsilon^{IJ}[\bar\lambda_I,\bar\lambda_J]\phi\Big)\ ,\end{aligned} \tag{0.1}$$

with $D, F, \bar F$ the auxiliary fields and $\lambda^I = (\lambda, \psi)$, as inherited from the $\mathcal{N}=1$ supermultiplets $\mathcal{V}$ and Φ.

- Going back to supersymmetry variations in Wess-Zumino gauge for the $\mathcal{N}=1$ in (7.64) of 'Supersymmetry and Supergravity' for the $\mathcal{N}=1$ vector multiplet $\mathcal{V}$, you should first derive the similar *Wess-Zumino gauge* $\mathcal{N}=1$ supersymmetry transformations on a chiral superfield Φ coupled to $\mathcal{V}$, which generalizes (4.10), (4.15) of these lectures to a charged chiral multiplet Φ coupled to $\mathcal{V}$.
- Then, show that the action (0.1) preserves $\mathcal{N}=2$ supersymmetry *in the Wess-Zumino gauge*, by first constructing these extended supersymmetry transformations explicitly. (Hint: You may use the fact that we know the Lagrangian preserves an WZ-gauge $\mathcal{N}=1$ supersymmetry and an $U(2)=SU(2)_R\times U(1)_r$ symmetry—see section 4.1. Then, use the extended R-symmetry to deduce the form of the $\mathcal{N}=2$ supersymmetry transformations, using the fact that the $\mathcal{N}=1$ supercharges do not commute with the $U(2)$ R-symmetry.)
- Check that these $\mathcal{N}=2$ supersymmetry variations realize the particular anticommutator:
$$\{\bar{\mathcal{Q}}_I, \bar{\mathcal{Q}}_J\} = 2\epsilon_{IJ}Z\ , \tag{0.2}$$
on the fields, with 'central charge' Z given by scalar field ϕ acting in the adjoint representation.

2. The Witten effect.

This question is essentially a short 'research project' on an important point which was not covered in the lectures. It will give you the opportunity to learn this beautiful subject on your own.

What is the Witten effect (for dyons)? Give a succinct explanation, including both general context and some technical details.

The best reference may be the original paper by Witten. (`http://cds.cern.ch/record/133312/files/197909065.pdf`) See also section 1 of: `https://arxiv.org/pdf/1312.2684.pdf` for more context.