Advanced Supersymmetry: Problem sheet 1

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Due by Friday, week 7 (March 5), 4pm.

1. Gauge anomalies and Standard Model.

The Standard Model of particle physics has a gauge group $G = SU(3) \times SU(2) \times U(1)_Y$. See section 9.1 of the lectures notes 'Supersymmetry & Supegravity.' Check equation (9.6), which shows that it is anomaly-free. (That is, give some more details showing that you understand how to go from eq.(9.5) to (9.6).)

2. Symmetries and 't Hooft anomalies in SQCD.

Consider SQCD, the 4d $\mathcal{N} = 1$ theory with one vector multiplet for a $SU(N_c)$ gauge group, N_f chirals $\Phi = Q$ in the fundamental representation of $SU(N_c)$, and N_f chirals $\Phi = \widetilde{Q}$ in the anti-fundamental representation.

2.a) Write down the classical global symmetries of the theory. What is the R-symmetry?

(2.b) Check that the $SU(N_c)$ gauge anomaly vanishes.

(2.c) Show that the axial symmetry $U(1)_A$ that assigns charges $A[Q] = A[\widetilde{Q}] = 1$ is anomalous—this is a chiral anomaly. How does the θ angle of SQCD transform under a $U(1)_A$ rotation?

(2.d) Is the R-symmetry you defined above anomalous?

(2.e) The flavor group of SQCD is $G_F = SU(N_f) \times SU(N_f) \times U(1)_B$—see Table 1 in the 'Advanced Supersymmetry' lectures. Compute all the possible 't Hooft anomalies for G_F, for any N_f and N_c.

3. $\mathcal{N} = 2$ SQCD.

(3.a) Write down the field content of $\mathcal{N} = 2$ SQCD, consisting of an $\mathcal{N} = 2$ vector multiplet and of N_f hypermultiplets, in terms of $\mathcal{N} = 1$ chiral superfields. [See Problem sheet 2 of 'Susy&Sugra' for a discussion of the 4d $\mathcal{N} = 2$ multiplets.]

(3.b) Write down the most general $\mathcal{N} = 1$ renormalizable Lagrangian with this field content. In particular, pay attention to possible superpotential terms.

(4.b) Write down the exact β-functions for τ and for the superpotential couplings. *In the absence of mass terms*, the superpotential is purely cubic in the $\mathcal{N} = 1$ chiral multiplets of the theory. Is a non-trivial fixed point (a zero of all the beta functions, for all coupling constants) possible at weak coupling, for some values of the parameters N_c and N_f? What are these values, if they exist?