Supersymmetry & Supergravity: Problem sheet 3

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Due by Friday, week 6 (February 28), 4pm.

The starred ([∗]) subquestions are optional and will not be marked.

1. Supersymmetric vacua: WZ models.

Analyse the vacuum structure of the following 4d $\mathcal{N}=1$ supersymmetric theories of chiral multiplets. You can assume a canonical Kähler potential. What is the energy of the vacuum, in each case?

(1.a) A theory of two chiral multiplets X and Y, with superpotential:

$$W = \lambda X^2 Y + \mu X^2 \ , \tag{0.1}$$

with λ, μ some non-zero coupling constants.

(1.b) A theory of three chiral multiplet X, Y and Z, with superpotential:

$$W = \alpha Y + \beta Y X^2 + \gamma X Z \ , \tag{0.2}$$

with $\alpha, \beta, \gamma \neq 0$. What happens when $\alpha = 0$?

(1.c) A theory of a single chiral multiplet X, with superpotential:

$$W = \alpha X + \frac{\beta}{X} \ , \tag{0.3}$$

with $\alpha, \beta \neq 0$.

2. Vector multiplets.

In the WZ gauge, the $U(1)$ vector multiplet reads:

$$V = -\theta\sigma^\mu\bar\theta A_\mu + i\theta\theta\bar\theta\bar\lambda - i\bar\theta\bar\theta\theta\lambda + \frac{1}{2}\theta\theta\bar\theta\bar\theta\, D \ . \tag{0.4}$$

The field-strength chiral multiplet is defined as:

$$\mathcal{W}_\alpha = -\frac{i}{4}\bar{\mathrm{D}}\bar{\mathrm{D}}\mathrm{D}_\alpha V \ . \tag{0.5}$$

(2.a) By explicit computation, show that the superfield $\mathcal{W}$ defined as in eq.(0.5) is given by:

$$\mathcal{W}_\beta = \lambda_\beta(z) - \theta^\alpha\Big((\sigma^{\mu\nu})_{\alpha\beta}F_{\mu\nu}(z) + i\epsilon_{\alpha\beta}D(z)\Big) + i\theta\theta(\sigma^\mu\partial_\mu\bar{\lambda}(z))_\beta \ , \qquad (0.6)$$

in terms of the chiral coordinate $z^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$.

(2.b) Compute the F-term Lagrangian:

$$\mathscr{L}_{\mathcal{W}^2} = -\frac{1}{2}\int d^2\theta \mathcal{W}^\alpha\mathcal{W}_\alpha \ , \qquad (0.7)$$

in field components.

(2.c) Compute the D-term Lagrangian:

$$\mathscr{L}_{\bar{\Phi}\Phi} = \int d^2\theta d^2\bar{\theta}\, \bar{\Phi}e^{-2V}\Phi \ , \qquad (0.8)$$

with Φ a chiral multiplet and V the vector multiplet in WZ gauge, in field components. You should find:

$$\mathscr{L}_{\bar{\Phi}\Phi} = \mathscr{L}_0 + A_\mu j^\mu + A_\mu A^\mu X + Y(D, \lambda, \bar{\lambda}) \ , \qquad (0.9)$$

for $\mathscr{L}_0$ the kinetic term without gauge field, some j^μ and X given in terms of the chiral superfield components, and a term Y that depends on D and the gaugino. Write the answer in terms of the gauge-covariant derivative.

(2.d) [∗] Generalise all the above superfield computations to the non-abelian case.

3. The SQED Lagrangian.

The supersymmetric version of QED, called SQED:

(3.a) Write down the Lagrangian, in superspace notation, for the minimal supersymmetric theory containing a $U(1)$ gauge field—the photon—and a single massive Dirac fermion of charge 1 coupled to the photon. Denote the gauge coupling by e^2.

(3.b) Write down the Lagrangian in components and in the WZ gauge (you may use the general results in the lecture notes). Integrate out the auxiliary fields F and D, to obtain a Lagrangian that only depends on the physical fields.

(3.c) Sketch the Feynman rules in this last formulation. In particular, write down the interaction vertices.

(3.d) Write down all the Feynman diagrams that contribute to the one-loop β function for SQED gauge coupling.

4. Supersymmetry breaking and goldstino.

In this problem, we explore F-term spontaneous supersymmetry breaking. Consider a general theory of n chiral multiplets Φ^i, with canonical Kähler potential and arbitrary superpotential $W(\Phi)$. Assume that supersymmetry is spontaneously broken, so that there exists a (classical) vacuum with a non-zero n-vector:

$$\bar{f}_i \equiv \frac{\partial W}{\partial \phi^i} \neq 0 \ , \tag{0.10}$$

where f^i is the VEV of the auxiliary field F^i on-shell.

(4.a) The classical vacuum is defined as:

$$\frac{\partial V_0}{\partial \phi^i} = 0 \ , \qquad \frac{\partial V_0}{\partial \bar{\phi}_i} = 0 \ . \tag{0.11}$$

Using the classical Lagrangian for this theory, show that eq.(0.10) implies the existence of a *massless fermion* in the vacuum. This is the *goldstino*, the analogue of a goldstone boson for spontaneously broken supersymmetry.

(4.b) Compute the masses of the fermionic and bosonic excitations just above the vacuum, and show that:

$$\mathrm{STr}(M^2) = 0 \ . \tag{0.12}$$

In other words, in terms of *mass eigenstates*, the sum of the boson masses squared *minus* the sum of the fermion masses squared vanishes.

(4.c) Illustrate these general results explicitly in the model of problem (1.b) with superpotential (0.2).