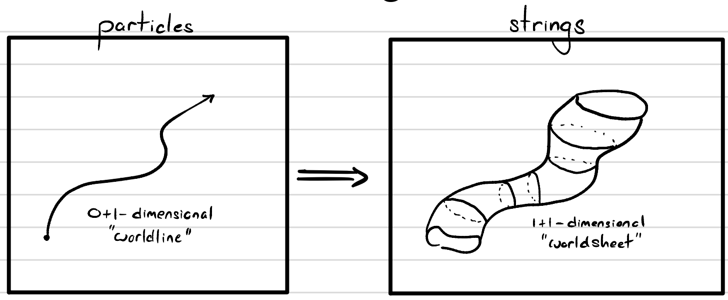
What is String Theory?

String theory is the theory of fundamental, quantum-mechanical strings.



Perturbative string theory (first discovered, best developed formulation)[+] is first-quantized[*], S-matrix theory.

[+]: many post-1990 developments give non-perturbative insights (duality, matrix models, ...). May see some in ST II.

[*]: string field theory is 2^{nd} quantized (but still perturbative) framework. Still not well understood in general. Won't use it.

Key features: consistently incorporates gravity (so a theory of quantum gravity)

is "unique" theory (this is a subtle issue; relates to dualities)

incorporates many interesting/phenomenologically relevant ingredients from QFT/particle physics.

- non-Abelian gauge symmetries w/chiral matter.
- spacetime supersymmetry
- "unification"

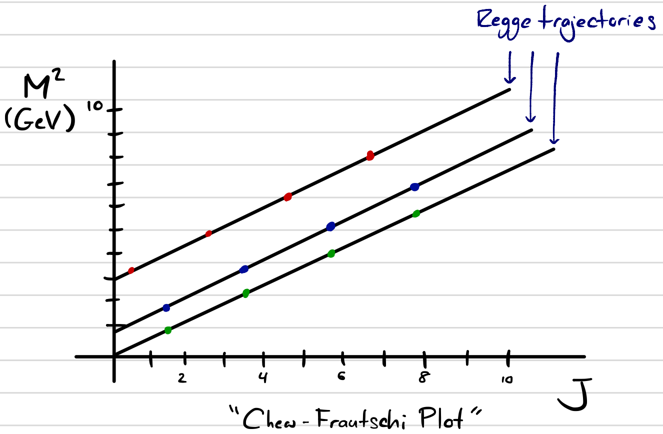
In this course, will develop bosonic string theory. This theory is missing some of the features mentioned above, and suffers from some serious defects/inconsistencies. However, it illustrates the key ideas/techniques in a relatively clean way.

If you continue on to ST II, you will learn superstring theory. This is the more exciting theory that has been considered as a candidate to some day describe our World.

Historical Motivation

Late 1950s/early 1960s : Hadronic particle zoo.

Many experimentally observed resonances w/ increasingly large invariant mass/spin.



"Chew - Frautschi Plot"

leading trajectory : $J = \alpha' M^2 + \alpha(0)$

$\alpha' \sim 1\,(\text{GeV})^{-2}$

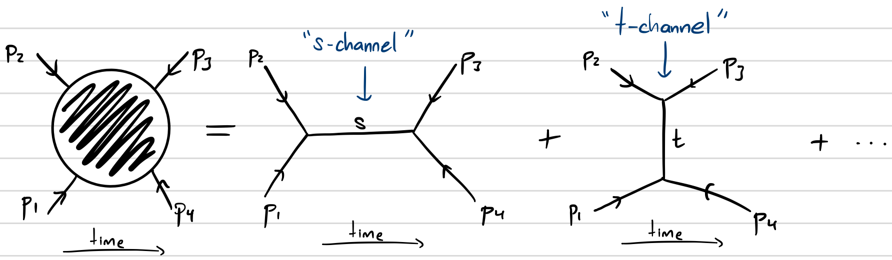
$\alpha(0)$: "Regge intercept"

lightest scalar : $M^2 = -\frac{\alpha(0)}{\alpha'}$

fixed $M^2 \to$ maximum spin $J(M^2)$

Problems: all resonances fundamental particles?
infinitely many?
can they be described using QFT?

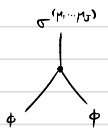
UV difficulties for high spin particles:



$$(-,+,+,+) \text{ signature } \begin{cases} s = -(p_1+p_2)^2 & (>0 \text{ for physical elastic scattering}) \\ t = -(p_1+p_4)^2 & (<0 \text{ for physical elastic scattering}) \\ u = -(p_1+p_3)^2 & (>0 \text{ for physical elastic scattering}) \end{cases} \quad (1,2) \to (3,4)$$

$$s + t + u = \sum_{i=1}^{4} m_i^2$$

Consider t-channel exchange of spin-J particle: $\delta A(s,t) \sim \frac{-g_J^2(-s)^J}{t - m_J^2}$ for fixed t, large (positive) s (Regge limit)

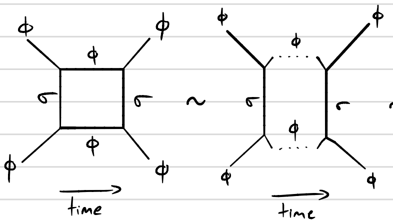


$$\boxed{\delta\mathcal{L} \sim g_J \left(\phi^* \overleftrightarrow{\partial}_{\mu_1} \overleftrightarrow{\partial}_{\mu_2} \cdots \overleftrightarrow{\partial}_{\mu_J} \phi\right) \sigma^{(\mu_1 \cdots \mu_J)}}$$

Historical Motivation

This "hard" UV behaviour is not what was observed in, say, pion scattering.

Even worse for loops:



$$\sim \int \frac{d^D p}{(2\pi)^D} \frac{p^{4J}}{(p^2)^4}$$

- safe for scalars
- logarithmic divergent for spin $J=1$
- badly divergent for $J>1$ $\Rightarrow$ not renormalizable

(general difficulties w/ theories of elementary higher spin particles.)

For finite sum, term w/ largest spin exchanged in t-channel dominates in Regge limit.

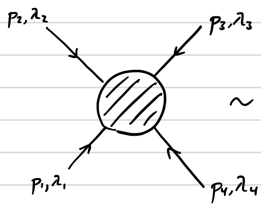
$$\sum_{J=0}^{J_{max}} \frac{-g_J^2(-s)^J}{t-M_J^2} \underset{\substack{s\to\infty \\ t \text{ fixed}}}{\sim} s^{J_{max}}$$

This is not necessarily the case if there are ∞'ly many exchange diagrams ...

$$\sum_{J=0}^{\infty} \frac{-g_J^2(-s)^J}{t-M_J^2} \sim \ ?$$

$$\boxed{\text{(Dolan-Horn-Schmid) Duality}}$$

With infinity of t-channel exchanges, perhaps s-channel poles arise automatically?



$$\sim \mathcal{A}(s,t)\, tr(\lambda_1\lambda_2\lambda_3\lambda_4) + (\text{other flavor orderings})$$

$$\mathcal{A}_t(s,t) \sim \sum_J \frac{-g_J^2(-s)^J}{t-m_J^2} \qquad \mathcal{A}_s(s,t) \sim \sum_J \frac{-g_J^2(-t)^J}{s-m_J^2}$$

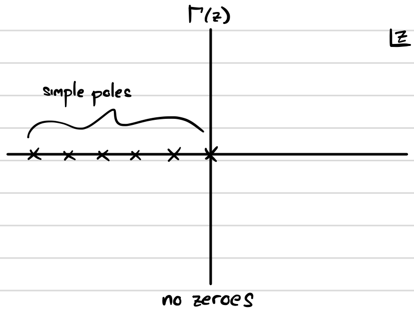
$$\text{QFT: } \mathcal{A}(s,t) = \mathcal{A}_t(s,t) + \mathcal{A}_s(s,t)$$

$$\text{"Dual Model": } \mathcal{A}(s,t) = \mathcal{A}_t(s,t) \overset{?}{=} \mathcal{A}_s(s,t)$$

Historical Motivation

Veneziano (1968): $\mathcal{A}(s,t) = \dfrac{\Gamma(-\alpha(s))\,\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))}$

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t}\,dt \longrightarrow$$



$$\alpha(s) = \underset{\text{Regge intercept}}{\alpha(0)} + \underset{\text{Regge slope}}{\alpha'} s$$

$$\begin{cases} \text{s-channel poles: } s = (-\alpha(0)+n)/\alpha' , & n = 0,1,2,\dots \\ \text{t-channel poles: } t = (-\alpha(0)+n)/\alpha' , & n=0,1,2,\dots \\ \text{double poles cancel w/denominator.} \end{cases}$$

Claim: $\mathcal{A}(s,t)$ satisfies DHS duality...

$$\mathcal{A}(s,t) = -\sum_{n=0}^{\infty} \frac{(\alpha(s)+1)(\alpha(s)+2)\dots(\alpha(s)+n)}{n!} \frac{1}{(\alpha(t)-n)}$$

$$= -\sum_{n=0}^{\infty} \frac{(\alpha(t)+1)(\alpha(t)+2)\dots(\alpha(t)+n)}{n!} \frac{1}{(\alpha(s)-n)}$$

From this representation, see that poles/resonances arrange on linear trajectories w/max spin $J=n$ @ $M^2 = \frac{-\alpha(0)+n}{\alpha'}$.

Historical Motivation

Virasoro (1969), Shapiro (1970) :

$$A(s,t,u) = \frac{\Gamma(-\alpha_c(s))\Gamma(-\alpha_c(t))\Gamma(-\alpha_c(u))}{\Gamma(-\alpha_c(s)-\alpha_c(t))\Gamma(-\alpha_c(t)-\alpha_c(u))\Gamma(-\alpha_c(u)-\alpha_c(s))}$$

$$\left[\begin{array}{l} \alpha_c(x) = \alpha(0) + \frac{\alpha' x}{4} \\ \alpha'(s+t+u) = -16\,\alpha(0) \end{array}\right]$$

$$\begin{cases} \text{s-channel poles: } s = 4(-\alpha(0)+n), & n = 0,1,2,\dots \\ \text{t-channel poles: } \quad " \\ \text{u-chamel poles: } \quad " \end{cases}$$

duality b/w all 3-channels.

max spin @ $m^2 = 4(-\alpha(0)+n)$ is $J = 2n$

Before long, people generalized to external particles other than lightest scalar.

$\Rightarrow$ Veneziano model

$\Rightarrow$ Virasoro-Shapiro model

High Energy Behaviour

Does this solve UV problems?

Veneziano in Regge limit ($s \gg 1$, $t<0$ fixed)

$$\mathcal{A}(s,t) \sim \Gamma(-\alpha(t))(-\alpha(s))^{\alpha(t)} \sim s^{\alpha(t)}$$

$\alpha(t) = \alpha(0) + \alpha' t$, so suppressed like $s^{-\alpha'|t|}$

(looks like Regge behaviour from exchange of single particle of negative spin $J(t) = -\alpha(0) - \alpha' t$, the "Regge-on")

Veneziano in fixed-angle, hard scattering limit ($s \gg 1$, t/s fixed)

$$\mathcal{A}(s,t) \sim [F(\theta_s)]^{-\alpha(s)}$$

real function greater than 1

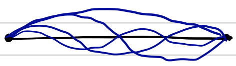
Falls off exponentially fast w/s !

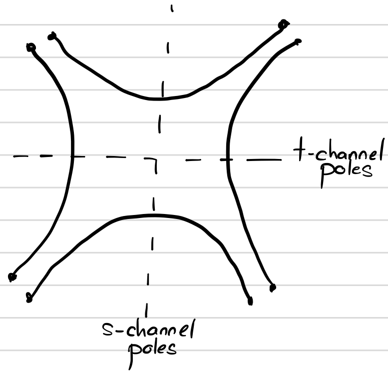
These are just formulas, but this suggests the theory behind them has good UV behaviour.
This will turn out to be true @ loop level as well...

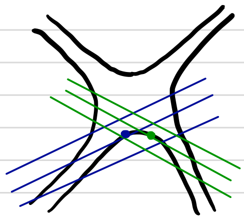
A Theory of Strings

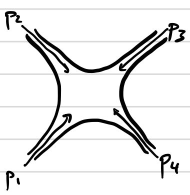
Theory behind Veneziano & Virasoro - Shapiro model turns out to be a theory where elementary particles are replaced by vibrating strings.

This gives heuristic justification for various good properties:

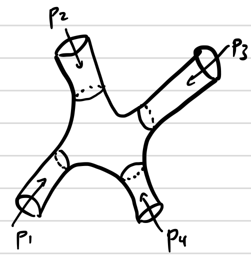
Spectrum: 

Duality: 

High-energy behaviour: 

$\mathcal{A}_{\text{veneziano}}(s,t)$: 

"open strings"

$\mathcal{A}_{\text{virasoro/shapiro}}(s,t,u)$: 

"closed strings"

Gravity & the String Scale

Veneziano / VS - amplitudes depend on two parameters : $\alpha(0)$ & α'

pure number

dimensionful, goes like $[\text{mass}]^{-2}$

Original idea : $\alpha' \sim (\text{GeV})^{-2}$ (nuclear physics energy scale)

$-\frac{\alpha(0)}{\alpha'}$ = mass^2 of lightest scalar = m_π^2

Problems :

- QCD is correct theory of strong nuclear force (solves UV problems differently).
- Unitarity of amplitude not manifest (require correct signs of residues @ poles when decomposed in Legendre polynomials)

"perturbative partial wave unitarity"

Eventually it was shown that one should have $\alpha(0) = 1$ ($m_\pi^2 < 0$?!)

⟶ Veneziano has pole for massless spin 1 (gauge boson ?)

⟶ VS has pole for massless spin 2 (graviton ?!??)

Suggests $\alpha' \sim (10^{19}\,\text{GeV})^{-2}$

J. Scherk & J. Schwarz (1974) : "Dual Models for Nonhadrons" (very readable!)

Show : "zero slope limit" (i.e. $\alpha' E^2 \ll 1$) of VS model gives tree-level gravity + scalar theory

$$S_{\text{zero-slope}} = -\int d^4x \sqrt{g} \left\{ \frac{1}{16\pi G_N} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right\} \quad \text{w/} \sqrt{8\pi G_N} = g_{str} \sqrt{\alpha'}$$