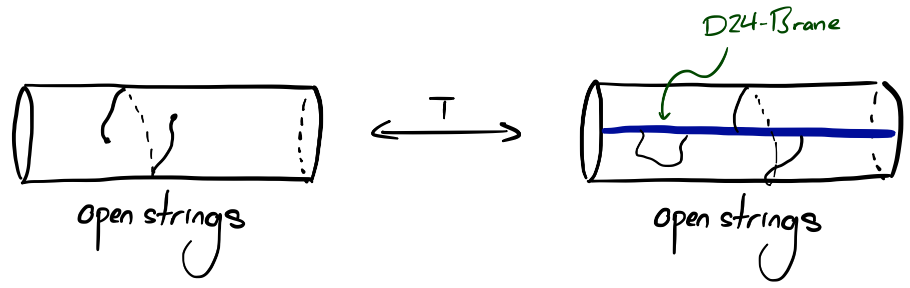
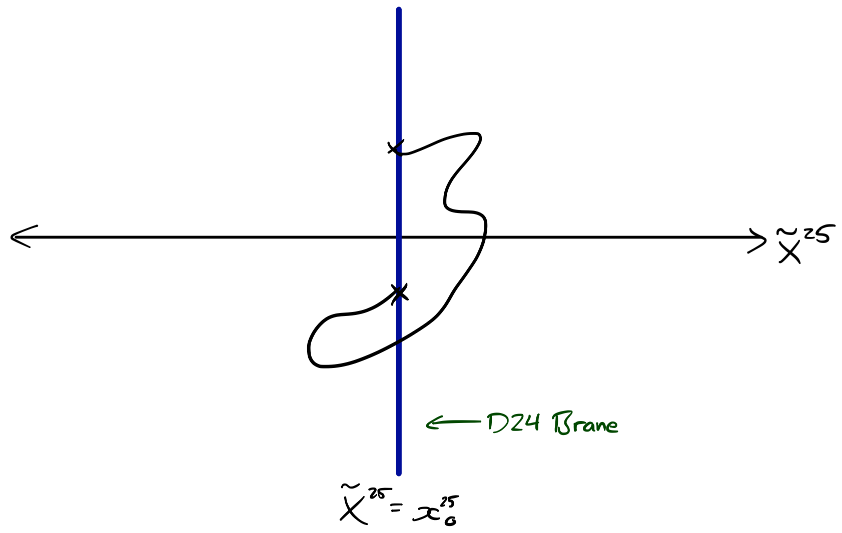
We'll finish up the course with some discussion of D-branes. Last time we introduced this terminology to refer to subspaces of spacetime upon which open strings can end. Arise in T-duality:



Let's look more closely at the effects of Dirichlet boundary conditions on the open string spectrum. For this purpose we'll eliminate the compactification complication.



The mode expansion for $\tilde{X}^{25}(\tau,\sigma)$ is much as before, but with $\cos \Rightarrow \sin$:

$$\tilde{X}^{25}(\tau,\sigma) = x_0^{25} + i\sum_{n\neq 0} \frac{\alpha_n^{25}}{n} e^{-in\tau} \sin(n\sigma)$$

$$X^i(\tau,\sigma) = x^i + \tau p^i + i\sum_{n\neq 0} \frac{\alpha^i}{n} e^{-in\tau}\cos(n\sigma) \qquad i = 0,1,\dots,24$$

The main difference is the absence of an α_0^{25}-mode. The L_m's are as before, but with α_0^{25} absent. Thus we have

$$L_0 - 1 = 0 \iff M_{25}^2 = -|\underline{p}|^2 = \underbrace{2N}_{\text{all 26 oscillators}} - 2$$

This looks like the "naive" dimensional reduction of the open string spectrum in the limit $R \to 0$ (so KK modes sent off to infinite mass). Let's look at massless physical states. A general level-one state is

$$|\xi,\eta;\underline{k}\rangle = (\underbrace{\xi\cdot\alpha_{-1}}_{\text{24+1-dim'l polarization vector}} + \underbrace{\eta}_{\text{spacetime scalar}}\alpha_{-1}^{25})|0;\underline{k}\rangle$$

The physical state condition is $\alpha_{+1}\cdot\alpha_0|\xi,\eta;\underline{k}\rangle = 0$ with 24+1-dim'l oscillators only b/c $\alpha_0^{25}\equiv 0$. Thus we have $\underline{k}\cdot\xi = 0$, η unconstrained. Similarly, the null states at level one are

$$L_{-1}|0;\underline{k}\rangle = \underline{k}\cdot\alpha_{-1}|0;\underline{k}\rangle \quad , \quad |\underline{k}|^2 = 0$$

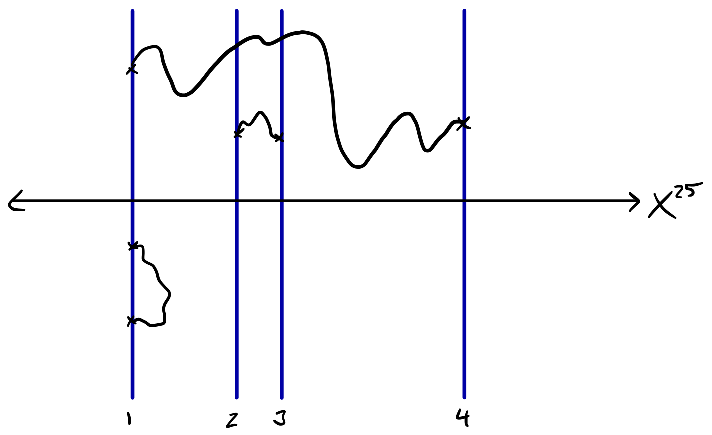
So we have a 24+1-dim'l photon, and an additional scalar field associated with $\alpha_{-1}^{25}|0;\underline{k}\rangle$. It turns out (we won't show it) that this scalar can be identified with fluctuations in the position of the D-brane in the transverse $\tilde{X}^{25}$ direction!

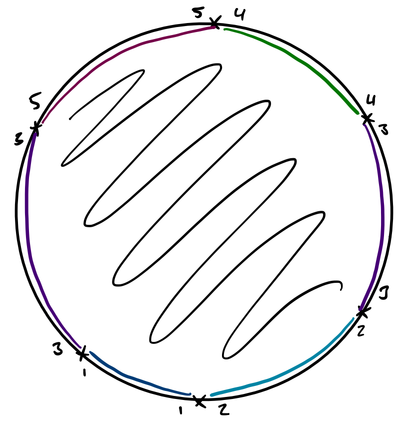
Comparing this with the T-dual picture, this scalar should be identified with the zero-mode of the A_{25} component of the gauge field, or more specifically, the integral

$$\int_0^{2\pi R} dx^{25} A_{25}(x^{25};\underline{x})$$

In both cases, it makes more sense to exponentiate these quantities to get single-valued quantities. For the gauge field, exponentiating gives the holonomy of the gauge connection around S_{25}^1. This is how we see the "position of the D-brane" in the T-dual picture.

We can also study systems of several D-branes, in which case strings interact according to shared boundaries:

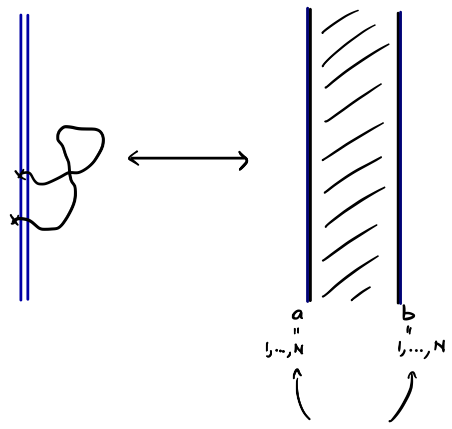




For stretched strings we have a kind of partial winding:

$$X^{25}_{ab} = x^{25}_a + \frac{x^{25}_b - x^{25}_a}{\pi}\sigma + i\sum_{n\neq 0}\frac{1}{n}\alpha^{25}_n e^{-in\tau}\sin(n\sigma) \;;\quad \alpha^{25}_0 = \left(\frac{x^{25}_b - x^{25}_a}{\pi}\right) \;;\quad M^2_{ab} = -p_ip^i = \left(\frac{x^{25}_b - x^{25}_a}{\pi}\right)^2 + 2(N-1)$$

The coincident limit is now very interesting!

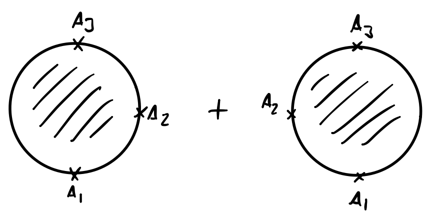


"Chan-Paton Factors"
(i.e., discrete d.o.f.'s associated to boundaries.)

All (ab) strings now have the same spectrum: $A_i, \varphi \longrightarrow (A_i)^a{}_b, \varphi^a{}_b$. The spectrum acquires a manifest $U(N)$ global symmetry, with all these open string states transforming in the adjoint representation. We can choose a basis for these states:

$$|\zeta;k;A\rangle = \sum_{a,b}(t^A)^a{}_b\,|\zeta;k;ab\rangle \qquad A = 1,\dots,N^2 \qquad t^A \text{ a basis for } u(N) \text{ (Hermitian)},\ Tr(t^At^B) = \delta^{AB}$$

Let's compute the 3pt. coupling of the massless vectors:



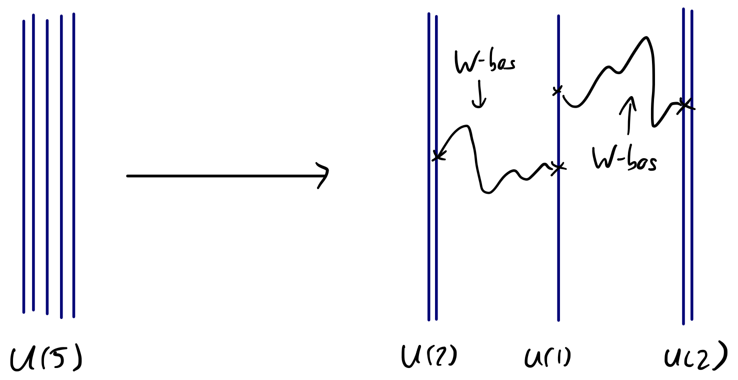
$$S(\zeta_1,k_1,A_1;\zeta_2,k_2,A_2;\zeta_3,k_3,A_3) \sim g_o\,\delta(k_1+k_2+k_3)\times\left\{\zeta_1\cdot k_{23}\,\zeta_2\cdot\zeta_3 + \zeta_2\cdot k_{31}\,\zeta_3\cdot\zeta_1 + \zeta_3\cdot k_{12}\,\zeta_1\cdot\zeta_2 + \frac{\alpha'}{2}\zeta_1\cdot k_{23}\,\zeta_2\cdot k_{31}\,\zeta_3\cdot k_{12}\right\}Tr\left(t^{A_1}[t^{A_2},t^{A_3}]\right)$$

This is the 3pt. vertex associated to the following action for a $U(N)$ non-Abelian gauge theory:

$$S = \underbrace{-\frac{1}{4}Tr(F_{\mu\nu}F^{\mu\nu})}_{\text{Yang-Mills}} - \underbrace{\frac{2i\alpha'}{3}Tr(F_\mu{}^\nu F_\nu{}^\omega F_\omega{}^\mu)}_{\alpha'\text{-correction}}$$

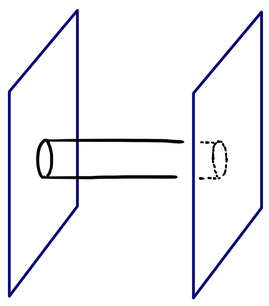
To derive this using β-functions, we need to introduce boundary couplings and boundary RG flows. Won't do it here!

The D-brane picture of non-Abelian gauge theory leads to a very beautiful picture of the Higgs mechanism.



In the stretched string sector, we have null states $L_{-1}|0,\underline{k};ab\rangle = \underbrace{|\underline{k},\underline{k};ab\rangle}_{\text{Longitudinal W's}} + \frac{X_{ba}}{\pi}\underbrace{|\eta,\underline{k};a,b\rangle}_{\text{Higgs Fields}}$: W's eat the Higgs! If we look at this in a T-dual frame, then this is "Wilson-line gauge-symmetry breaking", AKA, the Scherk-Schwarz mechanism.

A comment on D-brane dynamics: if these are dynamical objects, do we need to include them in our perturbative description of stringy physics? That could be tough! We can try to estimate the relevant mass scales associated to D-branes by determining their tension. A rough estimate comes from a closed string exchange b/w D-branes.



$\mathcal{A} \sim \underbrace{\kappa^2}_{\text{gravitational coupling} \sim g_o^2} \overbrace{\tau_p^2}^{\text{D-brane tension}} \sim (g_o)^{2-2g-h} = (g_o)^0$, so $\tau_p \sim 1/g_o^2$

So D-branes are "non-perturbatively heavy objects". On the other hand, what happens when $g_s \to \infty$? (~1995 "duality revolution" addressed this question).

Some final thoughts:

We've seen a lot of structure emerge automatically from quantizing strings

→ Gravity (quantized!)

→ Gauge fields

→ Duality

→ An S-matrix with good UV behavior

→ Spacetime dimensionality predicted

→ Nonperturbative "Brane" objects

If this is going to be a good framework for spacetime physics, we need some improvements

(A) → Tachyon Removal

(B) → Fermions

(C) → "Realistic" phenomenology?

→ Strong Coupling

→ Black hole physics

In string theory II, you will hear about the (simultaneous) resolution of (A) & (B). Supersymmetry (on the string worldsheet) is the key!