STRING THEORY II

Lecture I

OXFORD UNIVERSITY
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16H LECTURES, ASSESSMENT BY COURSE WORK. (4 SHEETS)

CLASSES: WED 11am BST WEEKS 3, 5, 7, 8

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PREREQUISITES: STRINGS I, (A)QFT, SUPERSYMMETRY, GR

REFERENCES:
- GREEN, SCHWARZ, WITTEN I & II (CUP).
- POLCHINSKI I & II (CUP).
- BLUMENHAGEN, LUEST, THEISEN (SPRINGER) (BLT)
  - ↳ MANY CONVENTIONS OF THE COURSE.

PLAN

1. FERMIONIC STRING: RNS (RAMOND - NEVEU - SCHWARZ) - STRING.
   $\hookrightarrow$ WORLDSHEET FERMIONS.
2. QUANTIZATION OF THE RNS.
3. $\boxed{10\,D}$ SUPERSTRINGS: TYPE IIA, IIB, I
4. D-BRANES IN SUPERSTRING THEORIES
   $\hookrightarrow$ ADDS A SUPER-YANG-MILLS SECTOR.
5. COMPACTIFICATIONS OF THE SUPERSTRINGS.
6. INTERACTIONS & EFFECTIVE ACTION.
7. COMPACTIFICATIONS ON CURVED BACKGROUNDS.
   $\hookrightarrow$ CALABI-YAU MANIFOLDS.

CHAPTER 1 CLASSICAL FERMIONIC STRING.

ACTION (GAUGE-FIXED)

$$S_B = -\frac{1}{4\pi\alpha'} \int d^2\sigma \; \partial^\beta_\alpha X^\mu \partial^\alpha X_\mu \quad \sqrt{-h}\, h_{\alpha\beta}$$

$\alpha = 0,1$ WORLDSHEET (WS) COORDINATE

$\mu = 0,\dots d-1$ TARGET SPACE

$$X^\mu : \Sigma \longrightarrow \mathbb{R}^{1,d-1}$$

S_B WAS OBTAINED BY GAUGE FIXING A STRING ACTION w/ WS METRIC $\underline{h_{\alpha\beta}}$: $T_{++} = T_{--} = 0$

PHYSICAL STATE CONDITIONS.

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2 OPTIONS: $\boxed{\text{1) WS FERMIONS} \Rightarrow \text{RNS-STRING.}}$

2) TARGET SPACE FERMIONS $\Rightarrow$ GREEN-SCHWARZ STRING.
(WS scalars).
REF: GSW I.

SPINORS IN 2D

$SO(1,1)$ CLIFFORD ALGEBRA w/ GENERATORS $\gamma^{\alpha}{}_{A}{}^{B}$

$\alpha = 0,1$ $\quad A, B = \pm$ SPINOR INDICES.

$$\{\gamma^\alpha, \gamma^\beta\} = 2\eta^{\alpha\beta}\mathbb{1}$$

2D MATRIX REP: $\gamma^0 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = i\sigma^2$

$\gamma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \sigma^1$

CHIRALITY OPERATOR: $\gamma = \gamma^0\gamma^1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$\Rightarrow \quad \psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} \qquad \psi_\pm \ \pm$ CHIRALITY.

2D SPECIALITY: WE CAN IMPOSE A REALITY CONDITION on ψ: "MAJORANA CONDITION"

$\bar{\psi} = \psi^T C \qquad C = \gamma^0 =$ CHARGE CONJUGATION.

$\Rightarrow \quad \psi_\pm$ CAN BE TAKEN TO BE REAL.

GAUGE-FIXED FERMIONIC STRING:

$\psi_{\pm}^{\mu}$ $\mu = 0, \ldots d-1$ SPACETIME VECTORS.

$$S_F = -\frac{1}{4\pi} \int d^2\sigma \; \underbrace{i \, \bar{\psi}^{\mu} \rho^{\alpha} \partial_{\alpha} \psi_{\mu}}$$

$$= -2 \left(\psi^{\mu}_{+} \partial_{-} \psi_{\mu +} + \psi^{\mu}_{-} \partial_{+} \psi_{\mu -} \right).$$

NB: $\psi^{+} = -\psi_{-}$

$\psi^{-} = \psi_{+}$

$\sigma_{\pm} = \tau \pm \sigma$

$$\Rightarrow \text{EOM:} \quad \begin{aligned} \partial_{-} \psi_{+} &= 0 \\ \partial_{+} \psi_{-} &= 0 \end{aligned} \quad \Rightarrow \quad \begin{aligned} &\psi_{+}(\sigma_{+}) \\ &\psi_{-}(\sigma_{-}). \end{aligned}$$

"quick analysis":

UN-GAUGE-FIXED RAMOND-NEVEU-SCHWARZ STRING

$\left(\begin{array}{c}\boxed{X^\mu(\sigma^\pm)} \\ \boxed{\psi^\mu_\pm}\end{array}\right)$ COUPLED TO WS METRIC $\left(\begin{array}{c}\boxed{h_{\alpha\beta}} \\ \boxed{\chi_\alpha}\end{array}\right)$ $\rightsquigarrow$ ZWEIBEIN $e_\alpha^{\ a}$.

GRAVITINO.

CLAIM: SUPERSYMMETRY TRANSFORMATION.

$$\epsilon = \begin{pmatrix}\epsilon_+ \\ \epsilon_-\end{pmatrix}$$

$$\epsilon_\pm(\sigma^\pm)$$

$$\delta_\epsilon X^\mu = i\bar{\epsilon}\psi^\mu$$

$$\delta_\epsilon \psi^\mu = \frac{1}{2}\gamma^\alpha\left(\sqrt{\frac{2}{\alpha'}}\partial_\alpha X^\mu - \frac{i}{2}\bar{\chi}_\alpha\psi^\mu\right)\epsilon.$$

$$\delta_\epsilon e_\alpha^{\ a} = \frac{i}{2}\bar{\epsilon}\gamma^a\chi_\alpha$$

$$\delta_\epsilon \chi_\alpha = 2D_\alpha\epsilon = 2\left(\partial_\alpha\epsilon - \frac{1}{2}\omega_\alpha\bar{\gamma}\epsilon\right).$$

$\omega_\alpha^{\ ab}$ = spin CONNECTION IN 2D.

$$\omega_\alpha = -\frac{1}{2}\omega_\alpha^{\ ab}\epsilon_{ab} = -\frac{1}{\sqrt{-h}}e_{\alpha a}\epsilon^{\beta\gamma}\partial_\beta e_\gamma^{\ a}$$
$$+\frac{i}{4}\bar{\chi}_\alpha\gamma^0\gamma^1\gamma^\beta\chi_\beta.$$

CLAIM: δ_ϵ LEAVES INVARIANT $S_{TOTAL} = S_B + S_F + S_\chi$:

$$S_B = -\frac{1}{8\pi}\int d^2\sigma \sqrt{-h}\left(\frac{2}{\alpha'} h^{\alpha\beta}\, \partial_\alpha X^\mu \partial_\beta X_\mu\right)$$

$$S_F = \frac{1}{4\pi}\int d^2\sigma \sqrt{-h}\left(-i\,\bar{\psi}^\mu \gamma^\alpha \partial_\alpha \psi_\mu\right)$$

$$S_\chi = \frac{i}{8\pi}\int d^2\sigma \sqrt{-h}\; \bar{\chi}_\alpha \gamma^\beta \gamma^\alpha \psi^\mu \left(\sqrt{\frac{2}{\alpha'}}\, \partial_\beta X_\mu - \frac{i}{4}\bar{\chi}_\beta \psi_\mu\right)$$

S_{TOTAL}: δ_ϵ (SUSY)

Weyl INVARIANCE

2d LORENTZ INVARIANCE

REPARAMETRIZATION INV.

$\Longrightarrow$ GAUGE FIX :

$$h_{\alpha\beta} = e^{\phi}\,\eta_{\alpha\beta}$$

$$\chi_\alpha = \gamma_\alpha \lambda$$

FIXED GRASSMANN VARIABLE.

"SUPER CONFORMAL GAUGE"

PROBLEM SHEET I:

CHECK LOCAL SUSY & gauge FIX TO

$$S_{RNS} = -\frac{1}{8\pi} \int d^2\sigma \left(\frac{2}{\alpha'} \partial_\alpha X^\mu \partial^\alpha X_\mu + 2i\, \bar{\psi}^\mu \rho^\alpha \partial_\alpha \psi_\mu \right)$$

SUSY:

$$\left.\begin{aligned} \sqrt{\tfrac{2}{\alpha'}}\, \delta_\epsilon X^\mu &= i \bar{\epsilon}\, \psi^\mu \\ \delta_\epsilon \psi^\mu &= \sqrt{\tfrac{2}{\alpha'}}\, \tfrac{1}{2} \rho^\alpha \partial_\alpha X^\mu \epsilon \end{aligned}\right\}$$

S_{RNS} IS INVARIANT IF $\underline{\gamma^\beta \gamma_\alpha D_\beta \epsilon = 0.}$

LIKE FOR THE BOSONIC STRING: GAUGE FIXING COMES AT THE COST OF IMPOSING THE PHYSICAL STATE CONDITION:

EM TENSOR: $\frac{\delta S}{\delta h_{\alpha\beta}} \Rightarrow$

$$T_{++} = -\frac{1}{\alpha'}(\partial_+ X)^2 - \frac{i}{2}\psi_+ \partial_+ \psi_+$$

$$T_{--} = -\frac{1}{\alpha'}(\partial_- X)^2 - \frac{i}{2}\psi_- \partial_- \psi_-$$

SUPER CURRENT: COMES FROM FIXING χ_α TO $\gamma_\alpha \lambda$

$$\frac{\delta S}{\delta \chi_\alpha}: \quad J_\pm = -\frac{1}{2}\sqrt{\frac{2}{\alpha'}}\, \psi_\pm^\mu \partial_\pm X_\mu$$

FERMIONIC CURRENT ON THE WS.

Need to impose:

CONSERVATION: $\partial_- T_{++} = 0$

$$\partial_+ T_{--} = 0$$

$$\partial_\pm J_\mp = 0.$$