STRING THEORY

II

LECTURE VII

OXFORD UNIVERSITY
MMATH PHYS TT 2020

D-BRANES:

SPACETIME:

C_p-FORMS IN THE RR-SECTOR OF THE TYPE II STRINGS:

$$\left.\begin{array}{ll} \text{IIA} & C_{2p+1} \\ \text{IIB} & C_{2p+2} \end{array}\right\}$$

COUPLE TO $D2p$ & $D(2p+1)$ BRANES WHICH ARE EXTENDED MEMBRANES OF DIM $2p+1$ & $2p+2$, RESPECTIVELY.

EFFECTIVE ACTION:

$$S_{DBI} = \underbrace{\boxed{T_p}}_{\text{Dp-BRANE TENSION}} \int_{W_{p+1}} d^{p+1}y \; e^{-\Phi} \sqrt{-\det(G + B + 2\pi F)}$$

G, B: PULLBACK FROM 10D TO W_{p+1} OF METRIC & B-FIELD.

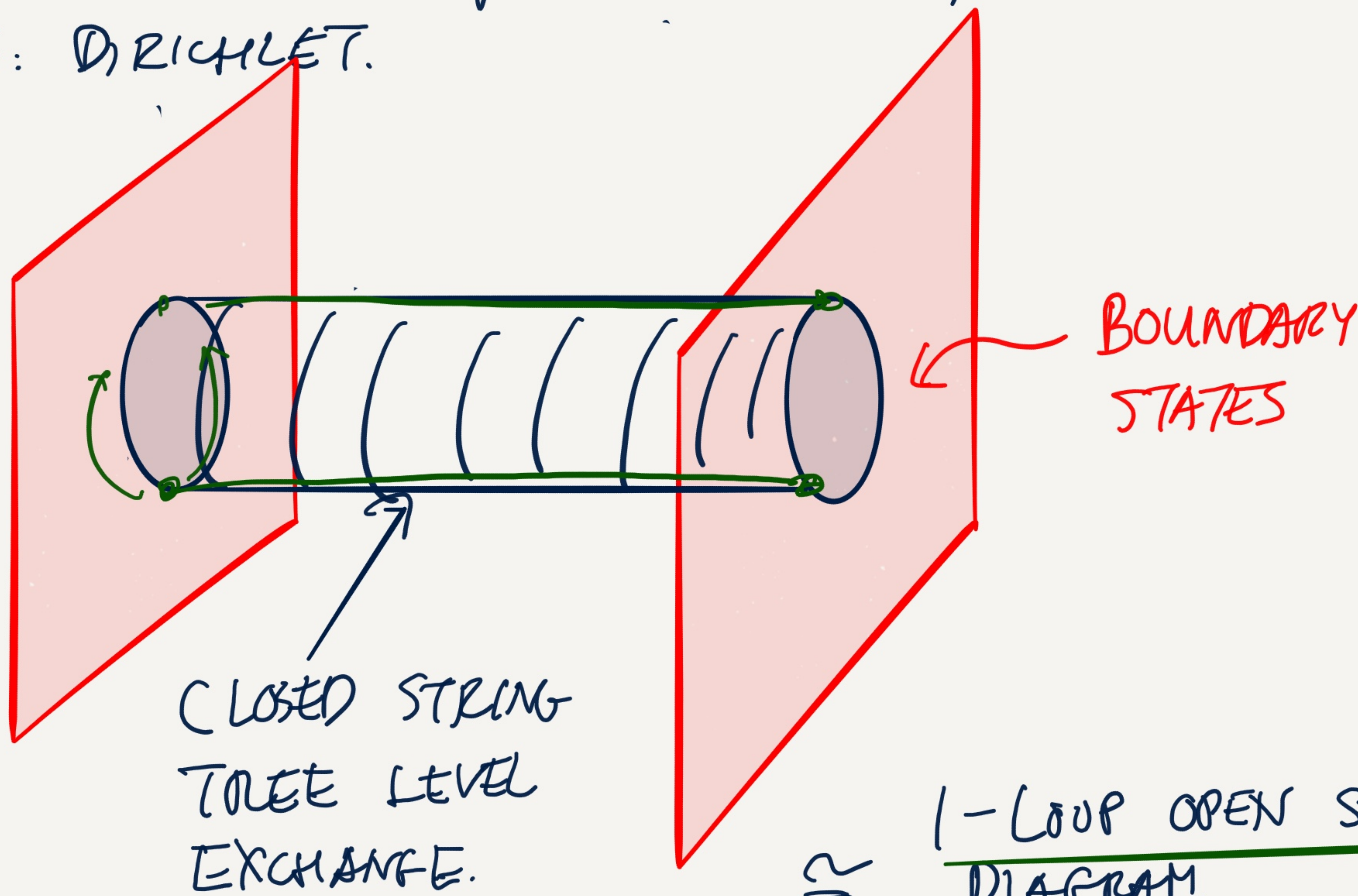
F: U(1) GAUGE FIELD.

$$S_{C_{p+1}} = \underbrace{\boxed{\mu_p}}_{\text{CHARGE UNDER RR-FORM FIELD.}} \int_{W_{p+1}} C_{p+1}$$

LAST LECTURE WE ARRIVED AT THE SAME CONCLUSION THAT IN TYPE B / A $\exists$ BOUNDARY CONDITIONS COMPATIBLE W/ SUPER-VIRASORO SYMMETRY & GSO-PROJECTION WHICH HAVE $|Dp\rangle\rangle$ $\quad p = 2k+1 \quad$ IIB

$p = 2k \quad$ IIA.

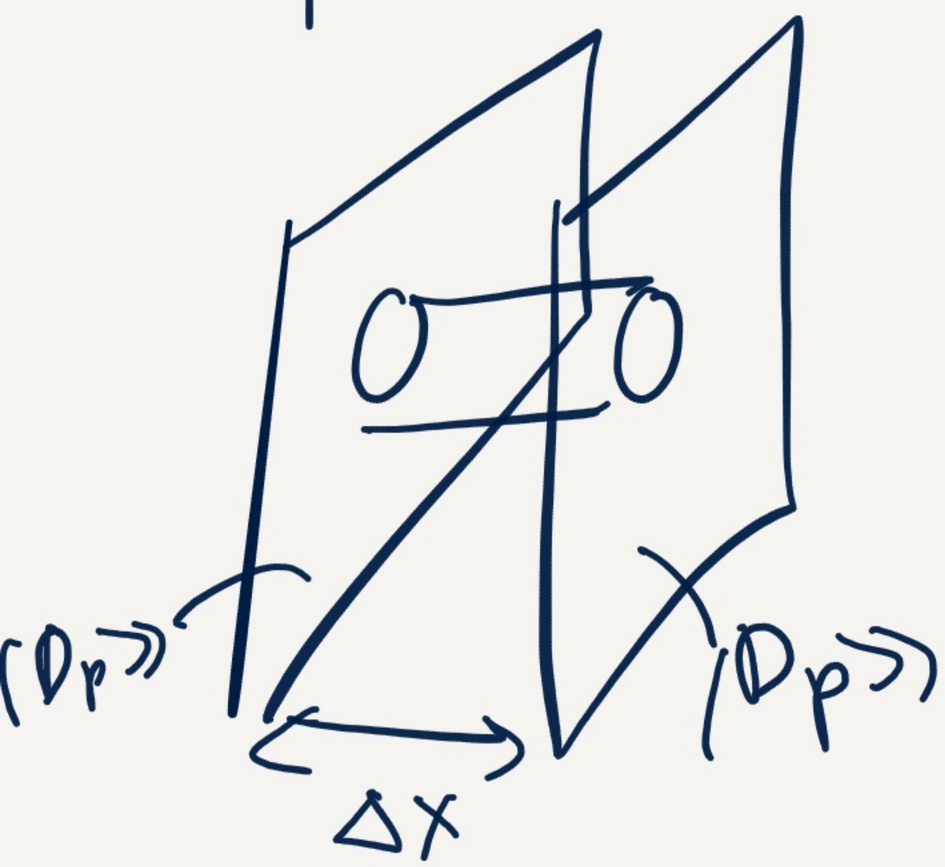
NEUMANN BC. ALONG $p+1$ SUBSPACE, & THE REMAINING DIRECTIONS: DIRICHLET.



$\simeq$ 1-LOOP OPEN STRING DIAGRAM.

FROM THIS CLOSED STRING TREE DIAGRAM WE COMPUTE THE COUPLING OF THE D-BRANES / BOUNDARY STATES TO $G_{\mu\nu}$ & Φ:

WORLDSHEET POINT OF VIEW.



$$\mathcal{A} = \int_0^\infty dl \, \langle\langle D_p | \, e^{-2\pi l H} \, | D_p \rangle\rangle$$

WE CAN EVALUATE THIS (LIKE IN PS2), & EXTRACT THE EXCHAGE OF LIGHT-CLOSED STRING STATES BY TAKING l large:

$$\mathcal{A} = \underbrace{\frac{\pi}{8} \left(\pi^2 4\alpha'\right)^{3-p}}_{} V_{p+1} \, (\underset{B}{16} - \underset{F}{16}) \, \frac{1}{\pi\alpha'} G_{9-p}(\Delta x_D) + \ldots$$

G = SCALAR GREEN'S FUN.

$\Rightarrow$ PREFACTOR DETERMINES COUPLING TO THE $G_{\mu\nu}$ & Φ.

FROM SPACETIME POINT OF VIEW:

G, Φ COUPLING IS SET BY THE DBI ACTION

COMPARING THESE IDENTIFIES THE TENSION OF Dp-BRANE AS:

$$\boxed{T_p = \frac{2\pi}{g_s (\alpha')^{\frac{p+1}{2}}}}$$

g_s = string coupling.

[Polchinski I & II].

LIKEWISE: COMPUTE THE RR-CHARGE POLCHINSKI HEP-TH/9510017:

$$\Rightarrow \quad \boxed{\mu_p = T_p}$$

$\Rightarrow \quad T \sim \frac{1}{g_s}$ KEY PROPERTY OF D-BRANES.

FOR OTHER MEMBRANES (SOLITONIC OBJECTS)

$T \sim \frac{1}{g_s^2}$

e.g. NS5-BRANE (6d MEMBRANE) COUPLES TO B_2 MAGNETICALLY.

CHARGE QUANTIZATION

"RECAP" FOR MAXWELL THEORY: $U(1)$ GAUGE FIELD A $\overset{4d}{\wedge}$

$F = dA$ MAXWELL EQ: VACUUM:

$$dF = 0$$

$$d{\star}F = 0$$

WE INTRODUCE SOURCES: ELECTRIC & MAGNETIC

$$dF = \star j_m$$

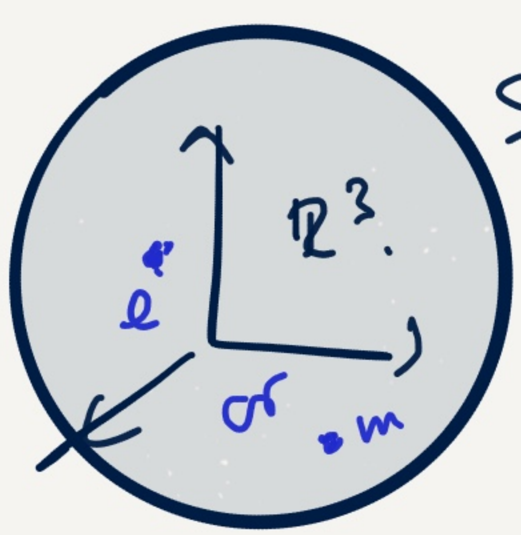
$$d{\star}F = \star j_e$$

j = 1-FORM $= j_\mu dx^\mu$

$\star_{4d}\, j$ = 3-FORM.

$$j = (\rho, \vec{j})$$

CHARGE DENSITY FOR EITHER ELECTRIC / MAGNETIC.



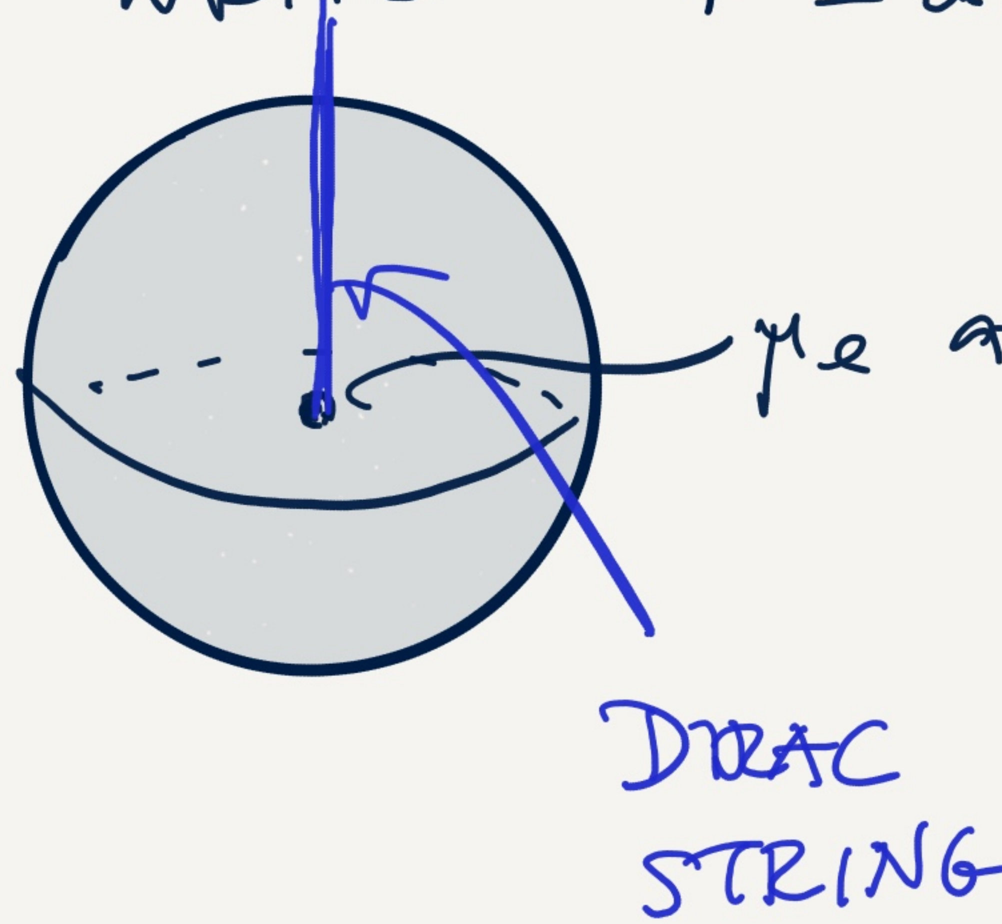
S^2.

$$\rho_i = \mu_i\, \delta^3(\underline{x})$$

$\partial\mathbb{R}^3 = S^2$ @ ∞.

$$\int_{\mathbb{R}^3} \rho_e = \int_{S^2} \star F = \mu_e \qquad \int \rho_m = \int_{S^2} F = \mu_m.$$

LOCALLY WE CAN WRITE $F = dA$ EXCEPT WHEN THERE IS A CHARGE DENSITY,



WE CANNOT DO SO GLOBALLY: $F = dA$ EVERYWHERE EXCEPT ALONG THE DIRAC STRING, THAT EXTENDS FROM THE CHARGE TO INFTY.

<u>IDEA:</u> WE NEED TO ENSURE THAT THE WAVEFUNCTION (WV) OF e IS WELL-DEFINED IN THE PRESENCE OF A MAGNETIC CHARGE m (MONOPOLE).

$$\psi(x) = \underbrace{e^{i q_e \int_C A}}_{\text{PHASE ALONG A PATH } C} \psi_0(x) \quad \text{WV of } e.$$

: C closed THIS SHOULD BE TRIVIAL.

C = CLOSED PATH

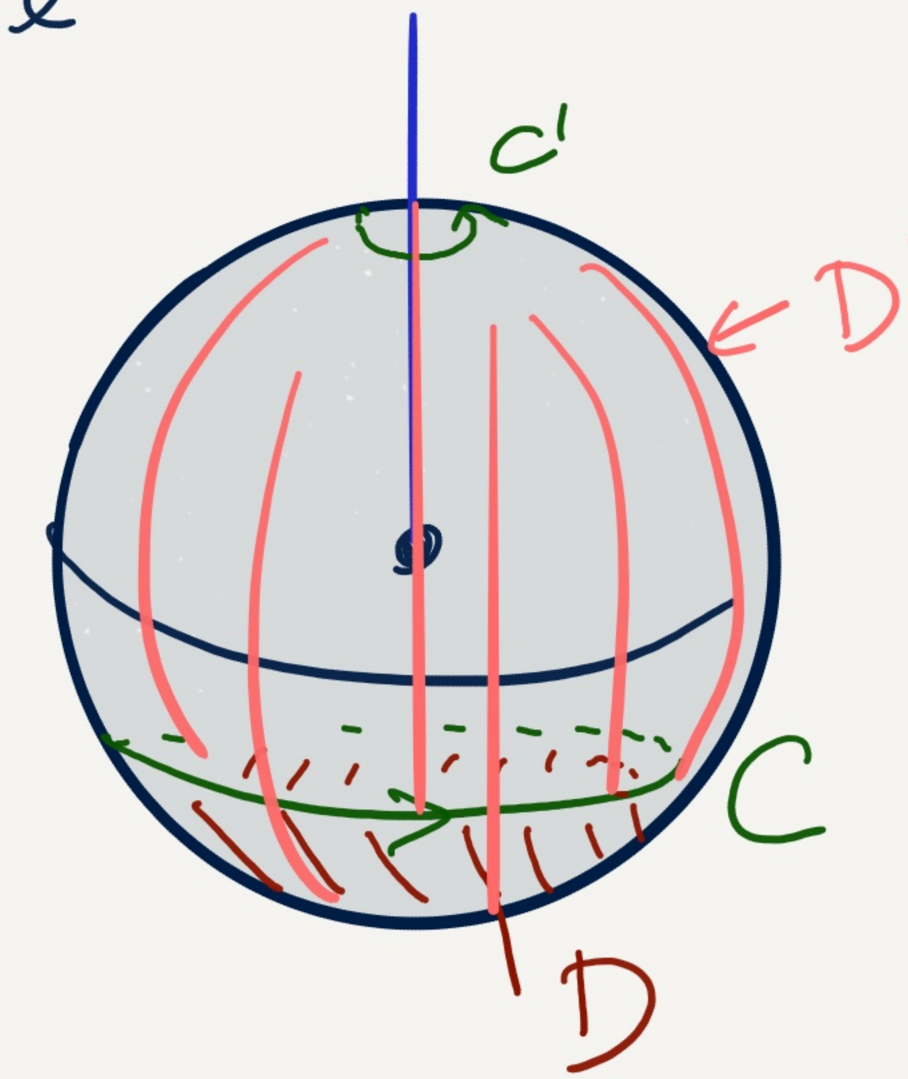
$$1 \overset{!}{=} e^{i\mu_e \oint_C A} = e^{i\mu_e \iint_D F} = e^{i\mu_e \mu_m}$$

$\partial D = C$

DISC

$C \to C'$

$D \to S^2$



$\Rightarrow \quad \mu_e \mu_m \in 2\pi\mathbb{Z}$ DIRAC CHARGE QUANTIZATION.

APPLY TO D-BRANES & RR - CHARGE QUANTIZATION:

$\boxed{D_p}$ COUPLES TO C_{p+1} FORM

$$F_{p+2} = d\,C_{p+1} \xrightarrow[\text{HODGE DUAL}]{*} F_{8-p} = d\,C_{7-p}$$

$$\boxed{D(6-p)}$$

$\Rightarrow$ D_p ELECTRIC & $D(6-p)$ IS MAGNETIC WRT C_{p+1} FORM.

REPEATING THE SAME ARGUMENT AS IN MAXWELL CASE:

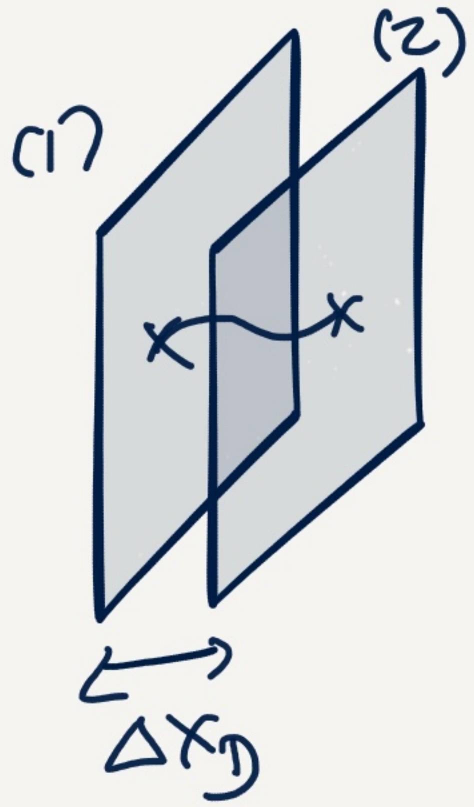
$$\int_{S^{p+2}} F_{p+2} = \mu_{6-p}$$

$$\Rightarrow \boxed{(\mu_{6-p} \cdot \mu_p) \in 2\pi\mathbb{Z}}$$

CHARGE QUANTIZATION FOR D-BRANE CHARGES.

OPEN STRINGS ON MULTIPLE D-BRANES

SPECTRUM OF (NEUMANN, NEUMANN) OPEN STRINGS (LECTURE 2).



MASS OF (D,D) STRING:

DISTANCE BETWEEN D-BRANES.

$$\alpha' M^2 = N^x + N^\psi + \frac{(\Delta x_D)^2}{4\pi^2\alpha'} + a$$

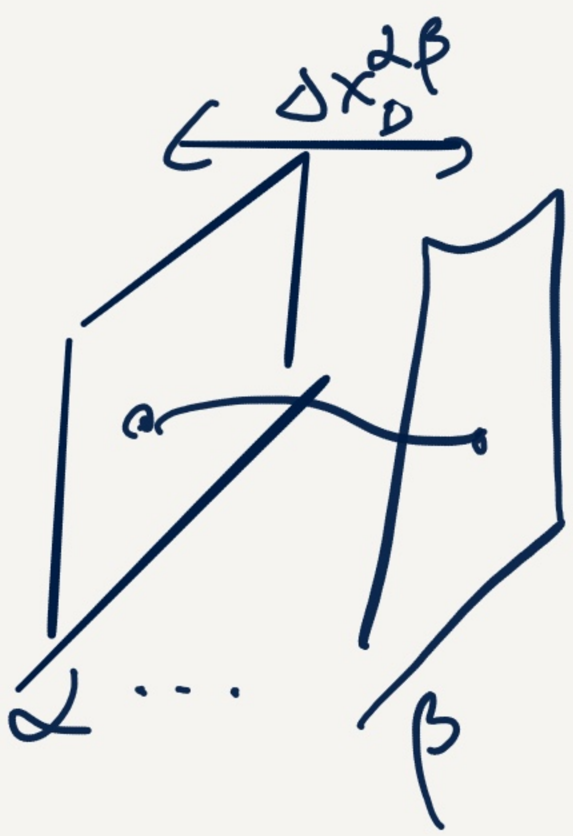
LOWEST STATES ARE MASSIVE: $\quad \text{Mass}^2 \sim (\Delta x_D) = \sum_{i=(D,D)\ \text{Directions.}} (x^i_{(1)} - x^i_{(2)})^2$

$\Delta x_D \to 0$: EXTRA LIGHT STATES.

N COINCIDENT D-BRANES: $\Delta x_D^{\alpha\beta} \to 0$

N^2 MASSLESS STATES.

$\Rightarrow$ $U(N)$ ADJOINT REPRESENTATION.



$$\lambda^a{}_{\alpha\beta} \qquad a = 1 \ldots N^2$$
$$\alpha, \beta = 1 \ldots N.$$

$\Rightarrow$ GROUND STATE: $\quad |a\rangle_{NS} = \sum_{\alpha,\beta=1}^{N} \lambda^a_{\alpha\beta}\, |0; \alpha\beta\rangle_{NS}$

Adj (U(N)).

NEXT LEVEL OF STATES: IN NS-SECTOR

$$\xi_i \, b^{i}_{-\frac{1}{2}} |a\rangle_{NS}$$

$i =$ DD Directions. $(0, \ldots p)$.

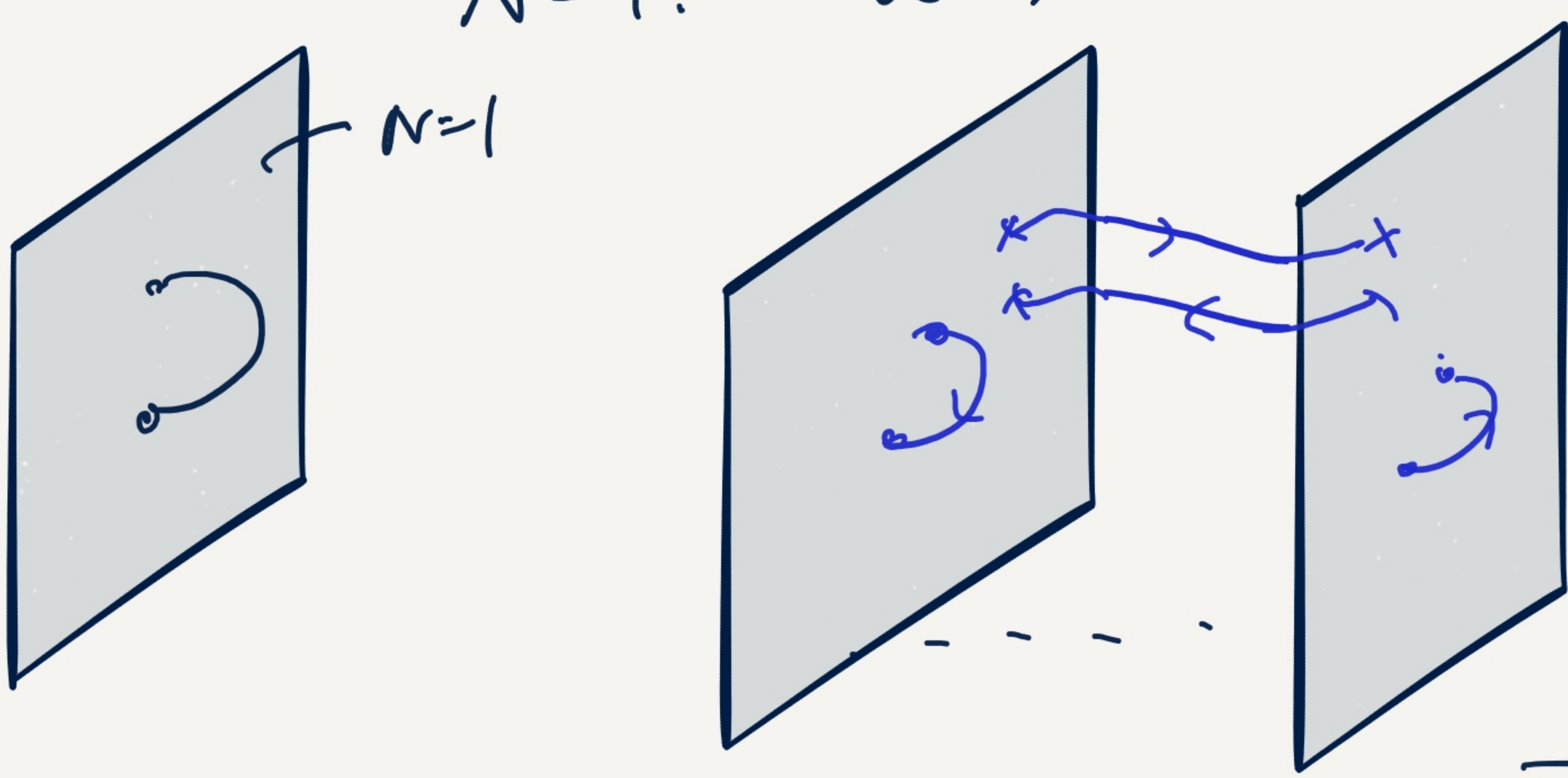
$U(N)$ Adjoint $\otimes$ VECTOR IN $SO(1,p)$

$\Rightarrow$ $U(N)$ GAUGE BOSON.

AFTER GSO PROJECTION (REMOVE VACUUM):

MASSLESS STATES ON A STACK OF N D-BRANES IS A $U(N)$ GAUGE FIELD.

$N=1$: $U(1)$ " " $\hat{=}$ F APPEARING IN THE S_{DBI}.



R-SECTOR:

$$|a\rangle_R = \sum_{\alpha,\beta=1}^{N} \lambda^{a}_{\alpha\beta} |0; \alpha\beta\rangle$$

R-SECTOR GROUND STATE.

$\Rightarrow$ GAUGINOS.

$\Rightarrow$ EFFECTIVE THEORY ON N COINCIDENT D_p: $(p+1)$dim $U(N)$ Super YM.

N Dp-BRANES: SUPER-YANG MILLS TH. IN p+1 DIM. (MAX SUSY) W/ GAUGE GROUP $U(N)$.

FAMOUS EX: N D3-BRANES: 4d $\mathcal{N}=4$ (MAX SUSY) SUPER YANG MILLS W/ $U(N)$ Gauge group.

$\rightsquigarrow$ STARTING POINT OF ~~A~~dS/CFT CORRESPONDENCE.

- EFFECTIVE ACTION CAN BE DERIVED AGAIN FROM STRING AMPLITUDES.
- OTHER GAUGE GROUPS:
  - ORIENTIFOLDS ($z \leftrightarrow \bar{z}$) $SO(N)$, $Sp(N)$
  - NON-PERTURBATIVE CONSTRUCTIONS "IIB" (F-THEORY) E_6, E_7, E_8 GAUGE GROUPS.

<u>NEXT LECTURE</u>: $\exists$ ALTERNATIVE WAY TO GET GAUGE THEORIES W/O D-BRANES: HETEROTIC STRING.

NEXT TIME WE WILL PREPARE THE CONSTRUCTION OF THE HETEROTIC STRING BY DISCUSSING COMPACTIFICATIONS ON TORI.

Bosonic STRING on T^{16} $\otimes$ RNS-STRING.

$26d - 16d \longleftrightarrow 10d.$

HET STRING IS THE FINAL MISSING PIECE IN THE LANDSCAPE OF 10D SUPERSTRINGS.