# STRING THEORY II

## Lecture IV

OXFORD UNIVERSITY
MMATHPHYS TT 2020.

# RECAP

- LIGHT CONE GAUGE QUANTIZATION OF THE RNS OPEN STRING W/ (N,N) B.C.
- SOLVED PHYSICAL STATE CONDITIONS FOR $X^I$ & $\psi^I$
- STATES : $\alpha' M^2 = N^X + N^\psi - a^{Total}_{NS/R}$

$$a^{Total}_{NS} = \frac{d-2}{16}$$

$$a^{Total}_{R} = 0$$

## NS SECTOR

$|k\rangle_{NS}$ TACHYON

$\underbrace{\zeta_i\, b^i_{-\frac{1}{2}}}\, |k\rangle_{NS}$ MASSLESS VECTOR $8_v$

$$a^{Total}_{NS} = \frac{d-2}{16} \Rightarrow \boxed{d=10}$$

## R SECTOR

$\{b^i_0, b^j_0\} = \delta^{ij}$ Clifford$(SO(d-2))$

$|a\rangle$ $8_s$ MASSLESS SPACETIME SPINORS.

$|\dot{a}\rangle$ $8_c$

$\Rightarrow$ PROBLEMS W/ SPECTRUM: TACHYON & NO SUSY.

TODAY: GSO PROJECTION $\Rightarrow$ TACHYON-FREE, SUSY SPECTRUM.

NAIVELY: PROJECTION BY $(-1)^F$   F = WORLDSHEET FERMION # OPERATOR.

THIS REMOVES TACHYON & ONE OF $8_s$ or $8_c$.
TO MOTIVATE & DERIVE THAT THIS CONSISTENT WE NEED TO GO ON SMALL DETOUR OF CHARACTERIZING THE RNS IN A 2D CFT LANGUAGE.

2D CFT APPROACH:

EUCLIDEAN WS $\leadsto$ WICK ROTATE: $z = \tau + i\sigma$   $\bar{z} = \tau - i\sigma$

BOSONIC STRING:
$$X^\mu(z,\bar{z})\, X^\nu(0,0) \sim -\frac{\alpha'}{2}\eta^{\mu\nu} \ln|z|^2.$$

OPERATOR PRODUCT EXPANSION (OPE)

FROM THE OPE WE CAN RECOVER THE OSCILLATOR ALGEBRA.

VIRASORO FIELD:

$T_{++}$ : $\quad T(z) = -\frac{1}{\alpha'} :\partial X^\mu \partial X_\mu:$

$T_{--}$ : $\quad \bar{T}(\bar{z}) = -\frac{1}{\alpha'} :\bar{\partial} X^\mu \bar{\partial} X_\mu:$

OPEs WILL BE DISCUSSED IN DETAIL IN CFT.

TO GET A WORKING UNDERSTANDING: 2-PT FN: 1 BOSON.

$$\langle \partial X(z)\, \partial X(w) \rangle \underset{\substack{\uparrow \\ \text{MODE EXP.}}}{=} -\frac{\alpha'}{2} \sum \underbrace{\langle 0 | \alpha_n \alpha_m | 0 \rangle}\, z^{-n-1}\, w^{-m-1}$$

$$= -\frac{\alpha'}{2} \sum_{n>0} n\, z^{-n-1}\, w^{n+1}$$

$n>0 \quad \alpha_n |0\rangle = 0$

$[\alpha_m, \alpha_n] = m\,\delta_{n,-m}.$

$$= -\frac{\alpha'}{2} \frac{1}{(z-w)^2} \qquad |z| > |w|.$$

$$\Rightarrow \quad \partial X(z)\, \partial X(w) \sim -\frac{\alpha'}{2} \frac{1}{(z-w)^2}.$$

$$X(z)\, X(w) \sim -\frac{\alpha'}{2} \log(z-w).$$

$\sim$ $\hat{=}$ UP TO REGULAR TERMS IN $z-w$.

LIKEWISE FROM THE OPE WE CAN REDERIVE THE OSCILLATOR ALGEBRA USING: E.G. EM TENSOR

$$T(z) = \sum_{n\in\mathbb{Z}} z^{-n-2} \underline{\underline{L_n}}$$


Cauchy $\Rightarrow$ $$L_n = \oint_0 \frac{dz}{2\pi i} z^{n+1} T(z).$$

$$\Rightarrow [L_m, L_n] = \oint_0 \frac{dw}{2\pi i} \oint_w \frac{dz}{2\pi i} z^{m+1} w^{n+1} \boxed{(T(z)\,T(w))} \; (*).$$

$\uparrow$ Difference of contour int. & contour def.



$\parallel$ OPE

$$\boxed{\frac{\frac{c}{2}}{(z-w)^4} + \frac{2\,T(w)}{(z-w)^2} + \frac{\partial T(w)}{(z-w)} + \text{TERMS REGULAR AS } z \to w.}$$

$$= \frac{c}{12} m(m^2-1)\delta_{n,-m} + (m-n) L_{m+n}.$$

NICE CFT EXPOSITIONS:

- DI MATHIEU, FRANCHESCO, SENECHAL "YELLOW BOOK"
- POLCHINSKI: I
- BLT

OPE W/ $T(z)$ DEFINES THE CONFORMAL WEIGHT:

$\mathcal{O}(z,\bar{z})$ local OPERATOR:

$$T(z)\,\mathcal{O}(0,0) \sim \frac{h}{z^2}\mathcal{O}(0,0) + \frac{1}{z}\partial\mathcal{O}(0,0) + (\text{REGULAR IN } z).$$

CONFORMAL WEIGHT.

$$L_0\left(\mathcal{O}(0,0)|\Omega\rangle\right) = h\left(\mathcal{O}(0,0)|\Omega\rangle\right).$$

<u>EXAMPLE:</u>

$\partial X(z)$ $\quad h=1 \quad \bar{h}=0$

$\partial$ " $\frac{\partial}{\partial z}$

$\bar{\partial}\bar{X}(\bar{z})$ $\quad h=0 \quad \bar{h}=1$

} GENERATE A $U(1)$ CURRENT ALGEBRA.

$:e^{ik\cdot X(z,\bar{z})}:$ $\quad h=\frac{\alpha' k^2}{4}=\bar{h}$

$:e^{ik\cdot X}:|\Omega\rangle = |k\rangle\otimes|k\rangle$

} MOMENTUM K state VERTEX OPERATOR.

<u>FIRST GOAL:</u> EXTEND THE 2D CFT APPROACH TO RNS STRING.

## FERMIONS

$$\psi^\mu(z)\,\psi^\nu(w) \sim \frac{\eta^{\mu\nu}}{z-w} \qquad \bar\partial\psi = 0$$

$$\tilde\psi^\mu(\bar z)\,\tilde\psi^\nu(\bar w) \sim \frac{\eta^{\mu\nu}}{\bar z-\bar w} \qquad \partial\tilde\psi = 0.$$

MODE EXPANSION:

$$\psi^\mu(z) = \sum_{r\in\mathbb{Z}+\phi} \frac{b_r^\mu}{z^{r+1/2}} \qquad \phi = \begin{cases} 0 & R \\ 1/2 & NS. \end{cases}$$

$$\tilde\psi^\mu(\bar z) = \sum_{r\in\mathbb{Z}+\phi} \frac{\tilde b_r^\mu}{\bar z^{r+1/2}}.$$

$$T_\psi(z) = -\frac{1}{2}\psi^\mu\partial\psi_\mu \Rightarrow \text{CONFORMAL WEIGHT}: 1/2.$$

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RNS:

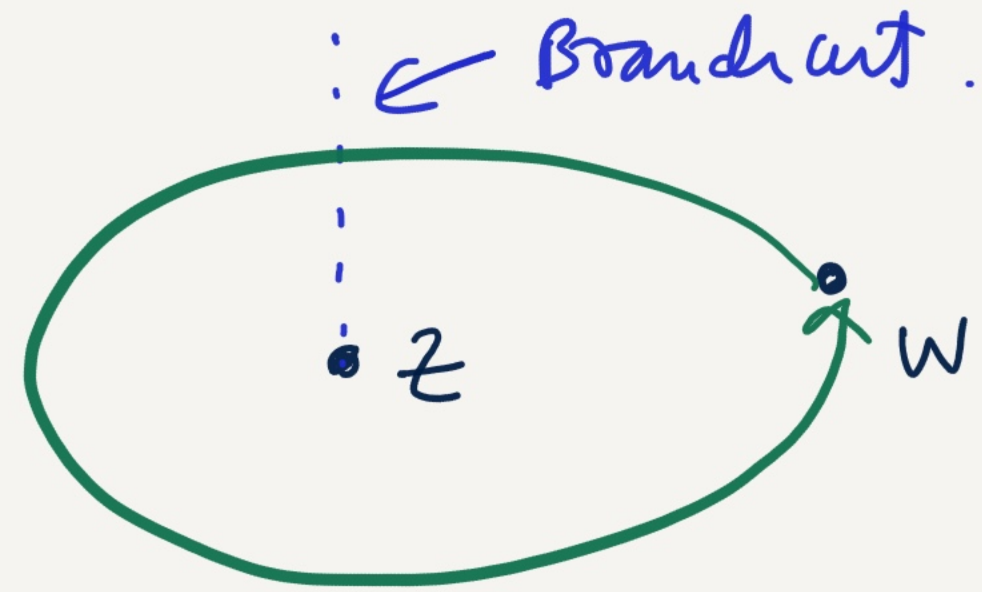
$$\begin{cases} T_X(z) + T_\psi(z) = T(z) = -\frac{1}{\alpha'}\partial X^\mu\partial X_\mu - \frac{1}{2}\psi^\mu\partial\psi_\mu. \\ J(z) = i\sqrt{\frac{2}{\alpha'}}\,\psi^\mu(z)\,\partial X_\mu(z) \\ \bar J(\bar z) = i\sqrt{\frac{2}{\alpha'}}\,\tilde\psi^\mu(\bar z)\,\bar\partial \bar X_\mu(\bar z). \end{cases}$$

NS SECTOR: $\phi = 1/2$ $\Rightarrow$ MODE EXPANSION IS IN INTEGER POWERS OF $z$.

R SECTOR: $\phi = 0$ $\Rightarrow$ $1/2$-INTEGER POWERS OF $z$!

$\Rightarrow$ BRANCH CUTS.

WE CAN PICK UP MONODROMIES AS WE ENCIRCLE FERMIONIC VERTEX OPERATORS. OPE SHOULD BE LOCAL:




$\Rightarrow$ CONSISTENCY OF THE OPE (LOCALITY) WILL IMPOSE CONSTRAINTS ON THE $\# \psi_s$ THAT WE CAN INSERT at $z$.

A SIMILAR SITUATION OCCURS IN THE BOSONIC STRING ON $S^1$:

$$V_{k,\bar k}(z,\bar z) = :e^{ikX(z)+i\bar k\bar X(\bar z)}:$$

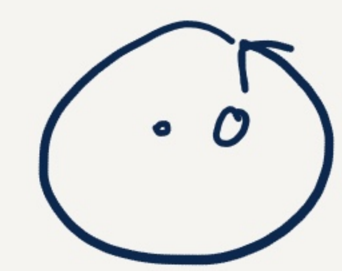
$(k,\bar k)$ MOMENTA: $V_{k,\bar k}$ CREATES MOMENTUM STATES.

OPE ALGEBRA:

$$V_{k,\bar k}(z,\bar z)\, V_{\ell,\bar \ell}(0,0) \sim \underbrace{z^{\frac{k\ell}{2}}\, \bar z^{\frac{\bar k\bar\ell}{2}}}_{} \, V_{k+\ell,\bar k+\bar\ell}(z,\bar z).$$

AGAIN: THIS CAN PICK UP A PHASE

$$(-1)^{(k\ell-\bar k\bar\ell)}$$



AS WE ENCIRCLE $V_{\ell\bar\ell}(0,0)$:

$\Rightarrow$ LOCALITY IMPLIES WE NEED TO ENSURE THAT

$$(-1)^{k\ell-\bar k\bar\ell} = 1$$

$$\Rightarrow \boxed{k\ell-\bar k\bar\ell \in 2\mathbb{Z}}$$

MOMENTUM QUANTIZATION $\rightsquigarrow$ MOMENTUM LATTICE.

LET'S RETURN TO THE RNS STRING: R-SECTOR.
WE CAN RELATE THIS PROBLEM TO THE PREVIOUS MOMENTUM OP. PROBLEM: <u>BOSONIZATION.</u>

PAIR OF MAYORANA-WEYL FERMIONS $\psi_1$ & $\psi_2$:

$$\Psi = \frac{\psi_1 + i\psi_2}{2} \qquad \bar{\Psi} = \frac{\psi_1 - i\psi_2}{2}$$

OPE $\Rightarrow$ $\boxed{\Psi(z)\,\bar{\Psi}(0) = \frac{1}{z}}$ $\qquad \Psi\,\Psi \sim 0$, $\quad \bar{\Psi}\,\bar{\Psi} \sim 0.$

$\simeq$ THUS OPE IS EQUIVALENT TO THE CHIRAL BOSON H OPE: $\quad H(z)\,H(0) \sim -\ln z$

$$\boxed{e^{i\underline{H(z)}}\; e^{-i\underline{H(0)}} \sim \frac{1}{z}}$$

ALL OTHERS VANISH.

THESE TWO CFTS (CHIRAL ALGEBRAS) ARE EQUIVALENT.

CHECK: VIRASORO AGREES.

$$\underbrace{\Psi,\ \bar{\Psi}}_{\text{2 MW FERMIONS}} \longleftrightarrow e^{iH},\ e^{-iH}$$

WE CAN USE THIS BOSONIZATION TO RESTRICT THE # OF FERMIONS THAT GIVE A LOCAL 2d CFT.

<u>DEF</u>: $(-1)^F$ $\quad$ $F =$ # WS FERMIONS.

<u>NS</u>: GROUND STATE TO BE ODD $\quad (-1)^F |0\rangle_{NS} = -|0\rangle_{NS}$

$$F = \sum_{r>0} b_{-r}^i b_r^i - 1$$

$\Rightarrow$ PROJECT ONTO $\boxed{(-1)^F = 1}$ NS.

<u>R</u>:

$$\left.\begin{aligned} (-1)^F |a\rangle_R &= |a\rangle_R \\ (-1)^F |\dot{a}\rangle_R &= -|\dot{a}\rangle_R \\ \{(-1)^F, b_r^i\} &= 0. \end{aligned}\right\} \quad (-1)^F = 16\, b_0^2 \dots b_0^9 \,(-1)^{\sum_{r>0} b_{-r}^i b_r^i}.$$

$\Rightarrow$ PROJECT ONTO $\boxed{(-1)^F = \pm 1}$ R.

# GSO PROJECTION

$\alpha = 1 - 2\phi \begin{cases} 1 & R \\ 0 & NS \end{cases}$ $\qquad F = \pm 1.$

CLOSED RNS – STRING:

$(\alpha, F ; \bar{\alpha}, \bar{F})$ $\qquad$ NS+ $\quad$ NS−
R+ $\quad$ R−

① NO BRANCHCUTS IN THE OPEs:

If $(\alpha_i, F_i)$ ARE PART OF THE SPECTRUM OF OPERATORS:

$$(-1)^{\underbrace{(F_1\alpha_2 - F_2\alpha_1 - \bar{F}_1\bar{\alpha}_2 + \bar{F}_2\bar{\alpha}_1)}_{\varphi}} \qquad \varphi \in 2\mathbb{Z}$$

② OPE NEEDS TO CLOSE.

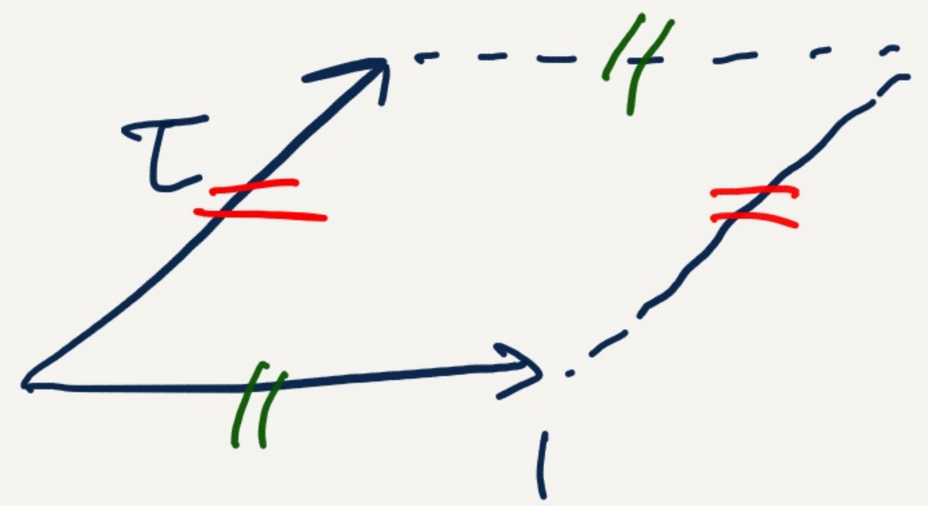
$(\alpha_i, F_i ; \bar{\alpha}_i, \bar{F}_i)$ $i = 1, 2$ IN THE SPECTRUM

$\Rightarrow (\alpha_1 + \alpha_2, F_1 + F_2 ; \bar{\alpha}_1 + \bar{\alpha}_2, \bar{F}_1 + \bar{F}_2)$ " " .

# ③ MODULAR INVARIANCE:

1-LOOP PARTITION FN OF THE CLOSED STRING.
"$T^2$- PARTITION FN".

$T^2$:




$\subseteq \mathbb{C}^2$

$\tau \in \mathbb{C}$.

$$T^2 \cong \mathbb{C}/\mathbb{Z} \oplus \tau\mathbb{Z}.$$

$$SL_2\mathbb{Z} \ni \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad \tau \longrightarrow \frac{a\tau+b}{c\tau+d}.$$

LEAVES THE $T^2$ PARTITION FN IS INVARIANT.

$\Rightarrow$ "1-LOOP PARTITION FN. NEEDS TO BE INVARIANT UNDER THIS $SL_2\mathbb{Z}$" $\hat{=}$ MODULAR INVARIANCE.

$\Rightarrow$ RNS: AT LEAST ONE LEFT & ONE RIGHT MOVING R-SECTOR.

CLOSED STRING SPECTRA SATISFYING THESE CONSTRAINTS ARE:

| | | | | |
|---|---|---|---|---|
| IIB | $(NS+, NS+)$ | $(R+, R+)$ | $(R+, NS+)$ | $(NS+, R+)$ |
| | $(-1)^F = (-1)^{\tilde{F}} = 1$ | | (CHIRAL THEORY) | |
| IIA | $(NS+, NS+)$ | $(R+, R-)$ | $(R+, NS+)$ | $(NS+, R-)$. |
| | $(-1)^F = 1$ | $(-1)^{\tilde{F}} = \begin{cases} 1 & NS \\ -1 & R. \end{cases}$ | (NONCHIRAL THEORY). | |

= BOSONIC FIELDS

= FERMIONIC FIELDS.