# STRING THEORY II

## LECTURE V

OXFORD UNIVERSITY
MMATH PHYS TT 2020

RECAP

# CLOSED STRING SPECTRA W/ GSO PROJECTION:

| | | | | |
|---|---|---|---|---|
| IIB | $\boxed{(NS+, NS+)}$ | $\boxed{(R+, R+)}$ | $\boxed{(R+, NS+)}$ | $\boxed{(NS+, R+)}$ |
| | $(-1)^F = (-1)^{\tilde{F}} = 1$ | | (CHIRAL THEORY) | |
| IIA | $\boxed{(NS+, NS+)}$ | $\boxed{(R+, R-)}$ | $\boxed{(R+, NS+)}$ | $\boxed{(NS+, R-)}$. |
| | $(-1)^F = 1$ | $(-1)^{\tilde{F}} = \begin{cases} 1 & NS \\ -1 & R. \end{cases}$ | | |
| | | | (NONCHIRAL THEORY). | |

$\boxed{\phantom{xxxx}}$ = BOSONIC FIELDS

$\boxed{\phantom{xxxx}}$ = FERMIONIC FIELDS.

SPECTRUM OF MASSLESS PHYSICAL STATES: 10 d: SO(8)

**BOSONIC**

$(NS+, NS+)$: $8_v \otimes 8_v = 1 \oplus 28 \oplus 35_v$

$\xi_i b^i_{-\frac{1}{2}} |0\rangle_{NS} \otimes \tilde{\xi}_i \tilde{b}^i_{-\frac{1}{2}} |\tilde{0}\rangle_{NS}$

- $1$: $\Phi$ scalar
- $28$: $[2]$ ANTISYMMETRIC 2-TENSOR B
- $35_v$: $(2)$ SYMMETRIC TRACELESS 2-TENSOR: G.

$(R+, R+)$: $8_s \otimes 8_s = 1 \oplus 28 \oplus 35_s$

- $1$: $[0]$ scalar $C_0$
- $28$: $[2]$ $C_2$
- $35_s$: $[4]_+$ $C_4$ ← SELF-DUAL.

$(R+, R-)$: $8_s \otimes 8_c = 8_v \oplus 56_v$

- $8_v$: $[1]$ $C_1$
- $56_v$: $[3]$ $C_3$.

IIB IIA.

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**FERMIONIC**

$(NS+, R+)$, $(R+, NS+)$: $8_v \otimes 8_s = 8_c \oplus 56_c$

- $8_c$: spin 1/2, $\lambda \hat{=}$ DILATINO.
- $56_c$: spin 3/2, $\psi \hat{=}$ GRAVITINO.

$(NS+, R-)$, $(R-, NS+)$: $8_v \otimes 8_c = 8_s \oplus 56_s$

- $8_s$: $\tilde{\lambda}$
- $56_s$: $\tilde{\psi}$.

- (NS, NS) SECTOR SPECTRUM: SIMILAR TO THE BOSONIC STRING.

  $\Rightarrow$ $\Phi$, B-FIELD, METRIC G.

- BOSONIC SECTOR: IN ADDITION $C_p$ ANTISYMMETRIC p-TENSOR.

  $\Rightarrow$ GENERALIZED GAUGE POTENTIALS.

  FIELD STREGTH $dC_p$ p+1-FORMS.

  $$C_p = C_{i_1 \dots i_p} dx^{i_1} \wedge \dots \wedge dx^{i_p}$$

  $\Rightarrow$ (R, R)-SECTOR FORM FIELDS WILL BE SEEN TO SOURCE D-BRANES.

  ['95 Polchinski]

- FERMIONS COMPLETE THE SPECTRUM TO A SUSY SPECTRUM ( G, $\phi$, $\lambda$, $\psi$ $\rightsquigarrow$ SUPERGRAVITY THEORY).

IIA & IIB SUPERSTRING THEORIES HAVE 2 10d SUSIES.

$\exists$ 1 SUSY THEORY: TYPE I STRING.

$I = IIB/\Omega$ $\quad$ $\Omega$ = WORLDSHEET PARITY: $\quad z \leftrightarrow \bar{z}$

THE TYPE I STRINGS ARE UNORIENTED (WS: MÖBIUS STRIP KLEIN BOTTLES)

$\Omega$:
$$\Omega \alpha_n^i \Omega^{-1} = \tilde{\alpha}_n^i$$
$$\Omega b_r^i \Omega^{-1} = e^{-2\pi i r} \tilde{b}_r^i$$

"EXCHANGES LEFT & RIGHT MOVERS".

I STRING IS OBTAINED BY PROJECTING IIB ONTO $\Omega = 1$

STATES: • (NS, NS) : $8_v \otimes 8_v \hat{=} b^i_{-1/2} |0\rangle_{NS} \otimes \tilde{b}^j_{-\frac{1}{2}} |\tilde{0}\rangle_{NS} S_i \tilde{S}_j$.

$\Rightarrow \Omega$ PROJECTS ONTO SYMMETRIC PART: $\Phi$, $\cancel{B}$, $G$ $\quad$ B ANTISYMM.

• (R, R): $8_s \otimes 8_s$ PROJECTS ONTO [2].

$\Rightarrow$ CLOSED STRING SECTOR: $\underset{\Phi}{[0]} \oplus \underset{G}{(2)} \oplus \underset{C_2}{[2]} \oplus 8_c \oplus 56_c$

THIS THEORY HAS ANOMALIES:

NEED TO ADD AN OPEN STRING SECTOR: $\oplus$ $\underline{(8_v \oplus 8_s)}$ $\underline{G = SO(32)}$

THERE IS ONE MORE THEORY IN 10D:
HETEROTIC STRING ($N=1$ SUSY).

$\rightsquigarrow$ COMPACTIFICATION CHAPTER.

$$\text{HET STRING} = \begin{pmatrix} \text{BOSONIC STRING ON} \\ T^{16} \end{pmatrix} \otimes (\text{R-NS STRING}).$$

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IIA, IIB $\rightsquigarrow$ $G$, $B$, $\Phi$, $C_p$, $N=2$ SUSY.

I, HET $\rightsquigarrow$ $N=1$ SUSY, THEY ALSO HAVE SUPER-YANG MILLS SECTOR.

IN TYPE I STRINGS THERE IS NO SUPER-YANG-MILLS IN THE CLOSED STRING SPECTRUM.

$\Rightarrow$ ONLY ONCE WE INCLUDE D-BRANES WILL WE HAVE YANG-MILLS SECTOR.

# PARTITION FUNCTION FOR THE RNS-STRING

"RECAP": BOSONIC STRING: PARTITION FUNCTION.

$$Z^B(q) = \sum_{n=0}^{\infty} \boxed{d_n}\, q^n$$

$d_n$ = degeneracy of states $\underline{\alpha' M^2 = n-1}$.

$$= \mathrm{Tr}_{\mathcal{H}^B_{\text{PHYSICAL STATES}}}\, q^{N^X}$$

$N^X$ = # operator.

$$= \prod_{n=1}^{\infty} \mathrm{Tr}\; q^{\alpha_{-n}\cdot\alpha_n}$$

$\alpha_i$ $i = 2 \dots d-1$

$$= \prod_{n=1}^{\infty} \left(\frac{1}{1-q^n}\right)^{d-2}$$

Dedekind Eta-function:

$$\eta(q) \equiv q^{1/24} \prod_{n=1}^{\infty} (1-q^n)$$

$$= \frac{q^{\frac{d-2}{24}}}{(\eta(q))^{d-2}}$$

# RNS-STRING:

$|0\rangle_{NS} \quad b_{-1/2}|0\rangle \quad b_{-\frac{3}{2}}|0\rangle \ldots$

NS-SECTOR: $(b_{-r})^2 = 0$

FOR 1 $b_r$: $$Z^F_{NS}(q) = \prod_{n=1}^{\infty} (1 + q^{n-\frac{1}{2}})$$

R-SECTOR: $$Z^F_R(q) = \prod_{n=1}^{\infty} (1 + q^n).$$

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SUPERSTRINGS W/ GSO PROJECTION:

GSO: INSERT $\frac{1}{2}((-1)^F + 1)$ - PROJECTION OPERATOR

INTO THE TRACE.

$$Z^{GSO}_R(q) = \mathrm{Tr}_{\mathcal{H}_R} \frac{1}{2}(1 + (-1)^F)\, q^{N_R^{total}}$$

W/ # OPERATOR. $$N_R^{total} = N^X + N^\psi = \sum_{n=0}^{\infty} \alpha^i_{-n} \alpha^i_n + n\, b^i_{-n} b^i_n$$

$$\frac{1}{2}\,\mathrm{Tr}_{\mathcal{H}_R}\, q^{N_R^{Total}} = 8 \prod_{n=1}^{\infty} \frac{(1+q^n)^8}{(1-q^n)^8}$$

$b^i_{-n}$

$\alpha^i_{-n}$

CONTRIBUTIONS FROM $\alpha^i_{-n}$ & $b^i_{-r}$ OSCILLATORS IN LC $(d=8)$

GROUND STATE HAS DEGENERACY 8.

$$\frac{1}{2}\,\mathrm{Tr}_{\mathcal{H}_R}\, (-1)^F q^N = 0$$

| | |
|---|---|
| $\vert a\rangle$ | $\vert \dot{a}\rangle$ |
| $(-1)^F = +1$ | $(-1)^F = -1$ |
| $b_{-1}\vert \dot{a}\rangle$ | $b_{-1}\vert a\rangle$ |
| $(-1)^F = +1$. | $(-1)^F = -1$ |

$$\boxed{Z_R^{GSO}(q)} = 8 \prod_{n=1}^{\infty} \left( \frac{(1+q^n)}{(1-q^n)} \right)^8 = \boxed{\frac{\theta_2(q)^4}{\eta^{12}(q)}}.$$

$$f_2(q^{1/2}) \equiv \sqrt{2}\, q^{\frac{1}{24}} \prod_{n=1}^{\infty} (1+q^n)$$

$$\equiv \sqrt{\frac{\theta_2(q)}{\eta(q)}}$$

NS- SECTOR W/ GSO.

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$$Z_{NS}^{GSO}(q) = \frac{1}{\underset{\text{GROUNDSTATE}}{q^{1/2}}} \operatorname{Tr} \frac{1}{2} (1+(-1)^F) q^{N_{NS}^{total}}$$

$N_{NS}^{Total} = N^X + N_{NS}^{\psi}$.

$$= \frac{1}{2q^{1/2}} \left( \prod_{n=1}^{\infty} \frac{(1+q^{n-1/2})^8 \ b\text{'s}}{(1-q^n)^8 \ \alpha\text{'s}} - \prod_{n=1}^{\infty} \frac{(1-q^{n-1/2})^8}{(1-q^n)^8} \right)$$

DEFINE: $f_{3/4}(q^{1/2}) \equiv \frac{1}{q^{1/48}} \prod_{n=1}^{\infty} (1 \pm q^{n-1/2}) \equiv \sqrt{\frac{\theta_{3/4}(q)}{\eta}}$

$$\Rightarrow Z_{NS}^{GSO}(q) = \frac{1}{\eta^{12}(q)} \left(\theta_3^4(q) - \theta_4^4(q)\right).$$

$$Z_R^{GSO}(q) = \frac{1}{\eta^{12}(q)} \theta_2^4(q).$$

GSO - PROJECTED OPEN RNS-STRING.

CLAIM: SUSY (SPACETIME) WOULD IMPLY:

$$\mathrm{Tr}_{\mathcal{H}^{GSO}} (-1)^{\mathcal{F}} q^N = 0$$ ✓

SPACETIME FERMION #.

$$\mathrm{LHS} = Z_{NS}(q) - Z_R(q) = \frac{1}{\eta(q)^{12}}\left(\theta_3^4 - \theta_4^4 - \theta_2^4\right)$$

SPACE-TIME BOSONS

FERMIONS.

$$= 0$$

JACOBI: $\theta_3^4 - \theta_4^4 - \theta_2^4 = 0$.

$\Rightarrow$ SPECTRUM SATISFIES BASIC CHECK OF SUSY!

NOTE: SIMILAR ARGUMENT FOR CLOSED STRINGS. IIA,B