STRING THEORY

II

LECTURE VI

OXFORD UNIVERSITY

MMATH PHYS TT 2020

SPECTRUM OF MASSLESS PHYSICAL STATES: 10 d: SO(8)

BOSONIC

$(NS+, NS+)$ $\quad 8_v \otimes 8_v = 1 \oplus 28 \oplus 35_v$

$\xi_i b^i_{-\frac{1}{2}} |0\rangle_{NS} \otimes \tilde{\xi}_i \tilde{b}^i_{-\frac{1}{2}} |\tilde{0}\rangle_{NS}$

Φ scalar; $[2]$ ANTISYMMETRIC 2-TENSOR B; (2) SYMMETRIC TRACELESS 2-TENSOR: G.

(R_+, R_+) $\quad 8_s \otimes 8_s = 1 \oplus 28 \oplus 35_s$

$[0]$ scalar $\boxed{C_0}$; $[2]$ $\boxed{C_2}$; $[4]_+$ $\boxed{C_4}$ SELF-DUAL.

(R_+, R_-) $\quad 8_s \otimes 8_c = 8_v \oplus 56_v$

$[1]$ $\boxed{C_1}$; $[3]$ $\boxed{C_3}$.

RR - p-FORMS

IIB IIA.

FERMIONIC

(NS_+, R_+)
(R_+, NS_+) $\quad 8_v \otimes 8_s = 8_c \oplus 56_c$

spin 1/2 $\lambda \hat{=}$ DILATINO. spin 3/2 $\psi \hat{=}$ GRAVITINO.

(NS_+, R_-)
(R_-, NS_+) $\quad 8_v \otimes 8_c = 8_s \oplus 56_s$

$\tilde{\lambda}$ $\quad \tilde{\psi}$.

EFFECTIVE ACTIONS: IIA & IIB STRINGS.

BOSONIC STRING:

[REF: VAN PROYEN & FREEDMAN: SUPERGRAVITY (CUP)]

THE EFFECTIVE ACTION OF Φ, B, G (MASSLESS MODES) WERE DETERMINED BY IMPOSING WEYL INVARIANCE:

ANOMALY " " : $T^a{}_a \sim \beta^G_{\mu\nu} \partial^a X^\mu \partial_a X^\nu + \dots$

$\Rightarrow$ SETTING $\beta^G = \beta^B = \beta^\Phi = 0 \iff$ SPACETIME EOM FOR Φ, B, G.

WE CAN DO THE SAME HERE (NS, NS) SECTOR WORKS IDENTICAL TO THE BOSONIC CASE:

$$S_{NSNS} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \, (-G)^{1/2} e^{-2\Phi} \left(R + 4\partial_\mu \Phi \partial^\mu \Phi - \frac{1}{2}|H_3|^2 \right)$$

10d Gravitational coupling constant.

$$|H_3|^2 = \frac{1}{3!} G^{M_1N_1} G^{M_2N_2} G^{M_3N_3} \times$$
$$\times (H_3)_{M_1M_2M_3} (H_3)_{N_1N_2N_3}$$

FOR (R,R) & (R,NS) & (NS,R) SECTOR FIELDS WE COULD PROCEED SIMILARLY, BUT WE WILL MAKE OF THE FACT THAT w/ $N=2$ SUSY (MEANING 2×16 dim spinors as supercharges)

THERE ARE TWO SUPERGRAVITY THEORIES: IIA, IIB.
[PREDATED THE STRING THEORIES].

BEFORE WE DISCUSS THESE, LET'S TALK ABOUT <u>P-FORMS</u>:
(DIFFERENTIAL FORMS: [NAKAHARA]).

RR-SECTOR WE HAVE P-FORMS:

$$C_p = \frac{1}{p!} \underbrace{C_{\mu_1 \dots \mu_p}}_{\text{TOTALLY ANTISYMM.}} \underline{dx^{\mu_1} \dots dx^{\mu_p}}.$$

$$(A_p \wedge B_q)_{\mu_1 \dots \mu_{p+q}} = \frac{(p+q)!}{p!\,q!} A_{[\mu_1 \dots \mu_p} B_{\mu_{p+1} \dots \mu_{p+q}]}$$

TOTAL ANTI-SYMMETRIZATION.

$p+q$ – FORM
"$\wedge$" WEDGE PRODUCT.

$$A_p \wedge B_q = (-1)^{pq} B_q \wedge A_p.$$

C_p WILL BE LIKE p-dim GAUGE POTENTIALS.

$dC_p = F_{p+1}$ (p+1)-FORM:

$$(dC_p)_{\mu_1 \ldots \mu_{p+1}} = (p+1)\, \partial_{[\mu_1} C_{\mu_2 \ldots \mu_{p+1}]}$$ EXTERIOR DERIVATIVE

HODGE STAR: $*$:

$$* A_{\mu_1 \ldots \mu_{d-p}} = \frac{1}{p!} \epsilon_{\mu_1 \ldots \mu_{d-p}}{}^{\nu_1 \ldots \nu_p} A_{\nu_1 \ldots \nu_p}.$$

$$(d-p)\text{-FORM} \xrightarrow{\;*\;} p\text{-FORM}.$$

NOTE: THIS DEPENDS ON THE METRIC!

THE ACTION FOR A p-FORM FIELD IS:

$$\int d^d x\, (-G)^{1/2}\, |F_p|^2 = \int d^d x \frac{(-G)^{1/2}}{p!} G^{M_1 N_1} \ldots G^{M_p N_p} \cdot F_{M_1 \ldots M_p} F_{N_1 \ldots N_p}.$$

$F_p = dA_{p-1}$

GAUGE TRANSFORMATION: $\delta A_{p-1} = d\lambda_{p-2}$.

EOMS:

$dF_p = 0$

$d * F_p = 0$ BIANCHI IDENTITY.

IIA & IIB ACTIONS ~~ARE~~ FIXED BY SUSY: BOSONIC PART IS

$$S_{IIA}^{RR} = -\frac{1}{4\kappa_{10}^2}\int d^{10}x\,(-G)^{1/2}\left(|F_2|^2 + |\tilde{F}_4|^2\right)$$

$$\tilde{F}_4 = dC_3 - C_1\wedge H_3$$

$H_3 = $ NSNS 3-form
$= dB_2$

C_1 & C_3 RR-FORMS.

$$S_{IIA}^{\text{CHERN-SIMONS}} = -\frac{1}{4\kappa_{10}^2}\int \underbrace{B_2\wedge F_4\wedge F_4}_{\text{10 FORM}}.$$

$$S_{IIB}^{RR} = -\frac{1}{4\kappa_{10}^2}\int d^{10}x\,(-G)^{1/2}\left(|F_1|^2 + |\tilde{F}_3|^2 + \frac{1}{2}|\tilde{F}_5|^2\right)$$

$$\tilde{F}_3 = \overset{=dC_2}{F_3} - C_0\wedge H_3$$

$H_3 = $ NSNS 3-FORM
$= dB_2$

$$\tilde{F}_5 = \overset{}{F_5} - \frac{1}{2}C_2\wedge H_3 + \frac{1}{2}B_2\wedge F_3$$

$F_5 = dC_4$

RECALL: C_4 WAS IN $[4]_+$ REP.

$$\boxed{\tilde{F}_5 = *\tilde{F}_5.}$$

SELF-DUALITY (SD) OF THE 5-FORM FIELD STRENGTH NEEDS TO BE IMPOSED IN ADDITION ($\nexists$ ACTION FOR A SD F_5).

$$S^{\text{CHERN-SIMONS}}_{IIB} = -\frac{1}{4\kappa_{10}^2}\int \underbrace{C_4 \wedge H_3 \wedge F_3}_{\text{10 FORM.}}$$

NSNS SECTOR: AS IN THE BOSONIC CASE:

G (EH ACTION)

Φ (SCALAR)

B_2 $\rightsquigarrow$ COUPLES TO THE STRING.

RR SECTOR: C_p & F_{p+1} FORM FIELDS.

$\rightsquigarrow$ WHAT'S THEIR PURPOSE?

$\rightsquigarrow$ D-BRANES $\qquad C_{p+1}$: Dp-BRANE

($p+1$ DIM OBJECT).

WE CAN WRITE DOWN

A "WORLD VOLUME" ACTION

D-BRANES:

Dp-BRANE COUPLES TO THE C_{p+1}-FORM:

$$\mu_p \int_{W_{p+1}} C_{p+1}$$

W_{p+1} = (p+1)-DIM WORLD-VOLUME

CHARGE OF THE Dp-BRANE UNDER THE RR-FORM FIELD.

"D-INSTANTON" C_0

IIB: $C_0, C_2, C_4^+ \longrightarrow D(\overbrace{2k+1}^{=p})$ D(-1), D1, D3, D5, D7

COUPLE TO THE C_p-FORMS EITHER ELECTRICALLY OR MAGNETICALLY.

NOTE: IIB THERE A D1-BRANE (D-STRING) & IT COUPLES TO C_2. VERY SIMILAR TO THE STRING ("FUNDAMENTAL STRING") F1. ∃ DUALITY THAT TREATS THESE TWO ON THE SAME FOOTING (S-DUALITY OF IIB-STRING).

IIA: $C_1, C_3 \longrightarrow D(\overbrace{2k}^{=p})$ D0, D2, D4, D6, D8

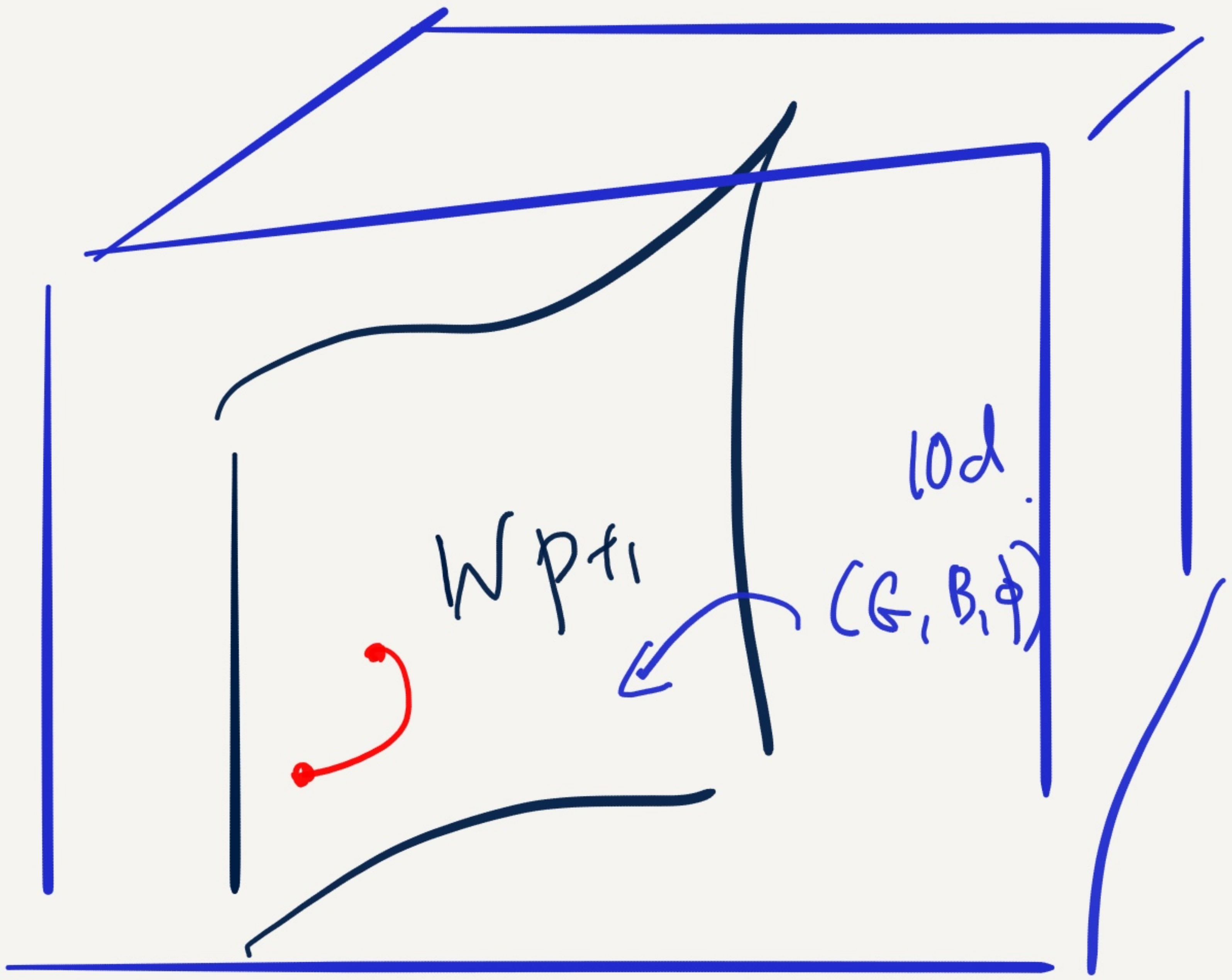
THE EFFECTIVE ACTION ON A D-BRANE RESEMBLES THAT OF THE NAMBU-GOTO ACTION OF THE STRING:

WORLD-VOLUME W_{p+1}: $\boxed{\text{DIRAC-BORN-INFELD ACTION}}$ (DBI)

$$S_{D_p} = -T_p \int_{W_{p+1}} d^{p+1}y \; e^{-\Phi} \sqrt{-\det(G + B + 2\pi\alpha' F)}$$

COORDINATES ON W_{p+1}

PULLBACK OF G (10d METRIC) ONTO W_{p+1}



T_p = TENSION OF THE Dp-BRANE.

F = U(1)-GAUGE FIELD.

FROM THE QUANTIZATION OF OPEN STRINGS:

$\rightsquigarrow$ (p+1)-DIM SUSY U(1) GAUGE TH.

D_p: 2 PARAMETERS: T_p & μ_p (TENSION & CHARGE)

IN THE NEXT LECTURE: WE STUDY D_p-BRANES FROM THE STRING POINT OF VIEW (I.E. 2D CFT).

$\rightsquigarrow$ BOUNDARY CFT ASSOCIATED TO THE RNS-STRING.
(IIA, IIB)

$\rightsquigarrow$ COMPUTE T_p, M_p.