STRING THEORY II

LECTURE VII

OXFORD UNIVERSITY

MMATH PHYS TT 2020

D-BRANES IN 2D CFT

$\Rightarrow$ BOUNDARY CFT (BCFT)

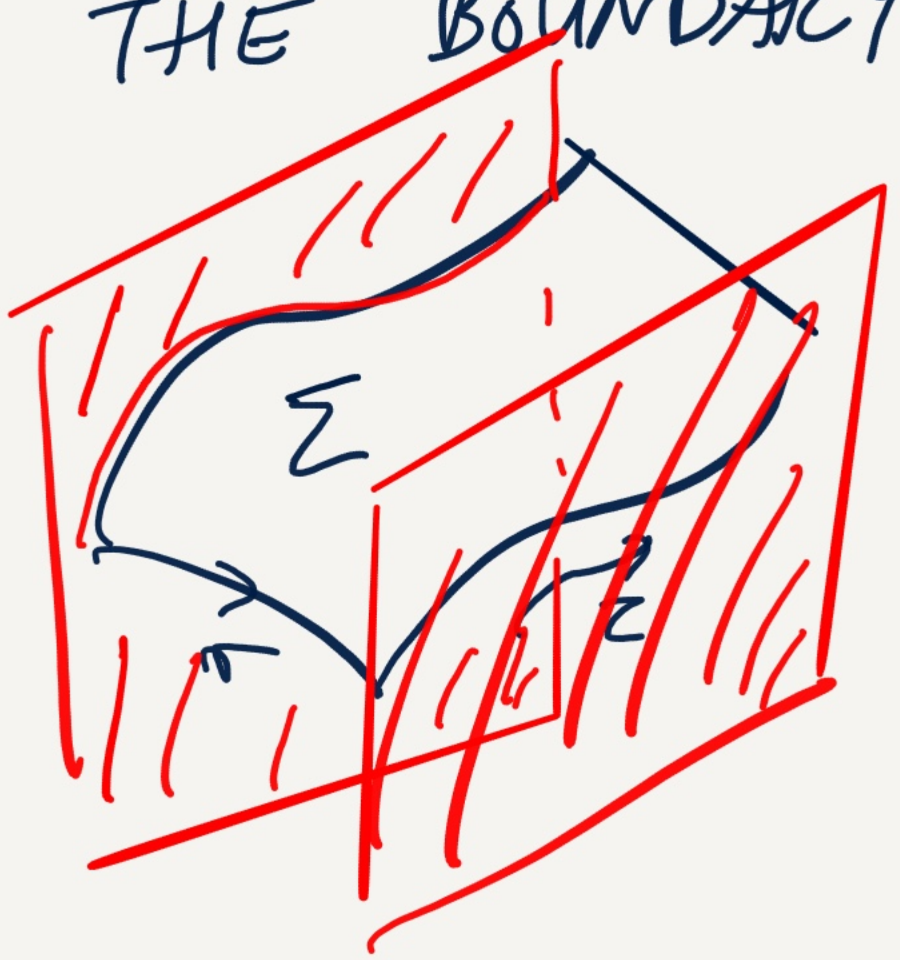
2D CFT LIKE THE ONE ON THE STRING WS
(HOWEVER BCFTs PLAY AN IMPORTANT ROLE ALSO
IN CONDENSED MATTER PHYSICS)

$\rightarrow$ BLT FOR AN EXPOSITION
IN STRINGS.

$\rightarrow$ COND-MAT: JOHN CARDY.

STUDY B.C. IN 2d CFTs:

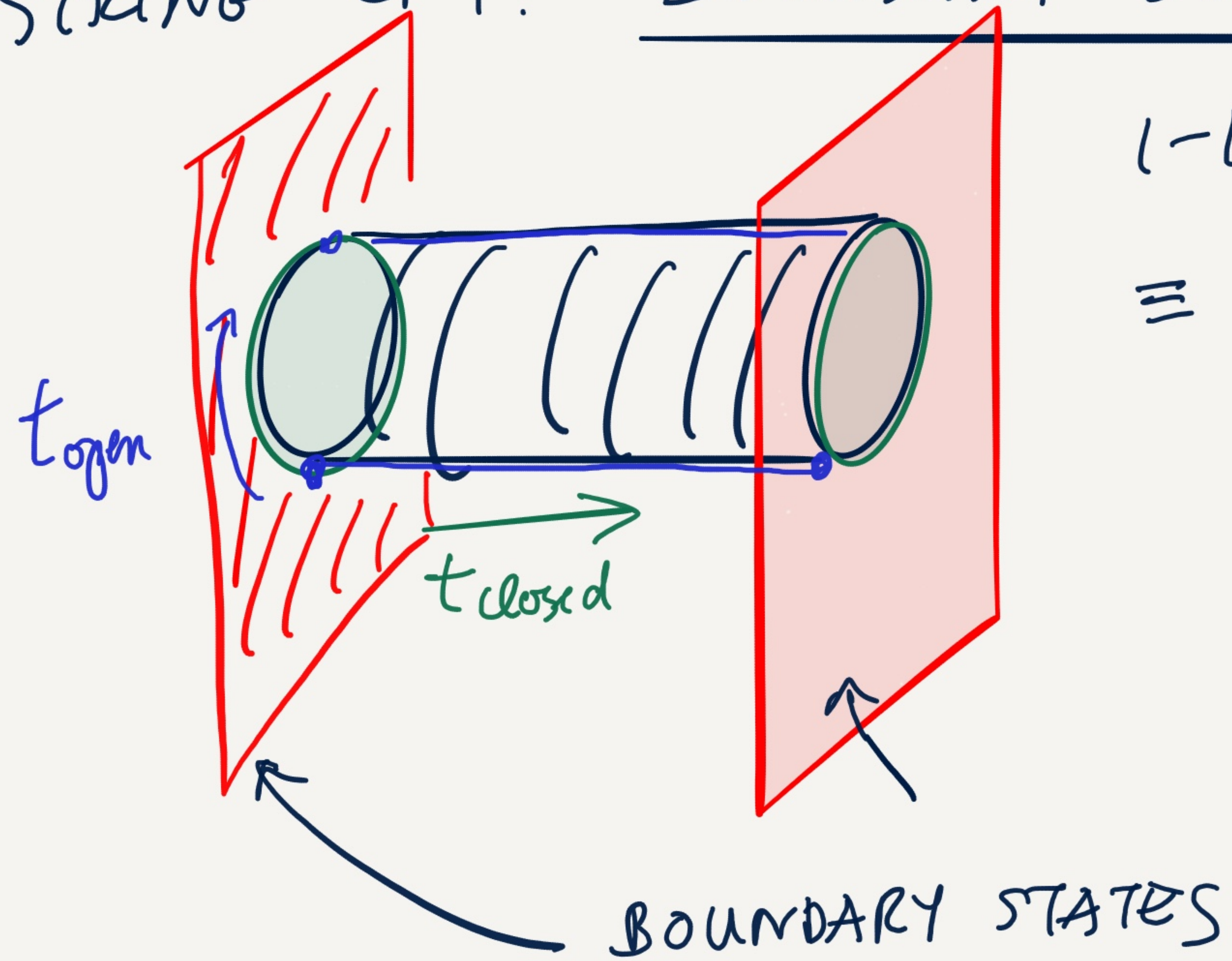
WE KNOW: <u>NEUMANN BC</u>: $\boxed{\partial_\sigma X^\mu = 0}$ AT THE BOUNDARY



DIRICHLET BC: $\boxed{X^\mu = x_0^\mu}$ AT THE BOUNDARY

THIS IS FROM AN OPEN STRING
POINT OF VIEW (SEE lecture 1 or 2).

A USEFUL POINT OF — IN PARTICULAR SINCE WE WANT TO STUDY B.C. IN THE CLOSED TYPE IIA & IIB THEORIES — IS TO CONSIDER THE FOLLOWING SETUP:
RATHER THAN STUDYING B.C. ON THE OPEN STRING WE WILL INTRODUCE NEW "STATES" IN THE 2d CLOSED STRING CFT: <u>BOUNDARY STATES</u>:



1-LOOP OPEN STRING DIAGRAM

$\equiv$ TREE-LEVEL closed STRING W/ BOUNDARY STATES INSERTED

$t_{open} \to \sigma_{closed}$
$\sigma_{open} \to t_{closed}$

WE CAN REWRITE THE B.C. IN TERMS OF CLOSED STRING DATA.

$$\boxed{\begin{array}{ll} \text{NEUMANN:} & \partial_\tau X^\mu_{closed} \, |B, N\rangle\rangle = 0 \\ \text{DIRICHLET:} & X^\mu_{closed} \, |B, D\rangle\rangle = x_0^\mu \, |B, D\rangle\rangle \end{array}}$$

$|B, N\rangle\rangle$ & $|B, D\rangle\rangle$ ARE THE BOUNDARY STATES IN TERMS OF THE MODE EXPANSION. (BOSONIC STRING).

NEUMANN: $(\alpha_n^{\mu} + \tilde{\alpha}_{-n}^{\mu})\, |B, N\rangle\rangle = 0$ $\quad \mu = 0 \ldots \underline{p}$

Relates LEFT & RIGHT MOVERS.

$\Rightarrow |B, N\rangle\rangle$ IDENTIFIES L/R MOVERS.

DIRICHLET: $(\alpha_n^{\mu} - \tilde{\alpha}_{-n}^{\mu})\, |B, D\rangle\rangle = 0$ $\quad \mu = p+1, \ldots D$

D = spacetime Dim.

USING THE REP OF THE VIRASORO ALGEBRA IN TERMS OF THE α & $\tilde{\alpha}$:

$$\Rightarrow \boxed{(L_n - \tilde{L}_{-n})\, |B, {}^{N}_{D}\rangle\rangle = 0}$$

INDENTIFIES THE LEFT & RIGHT VIRASORO ALGEBRAS.

BOUNDARY STATES THAT SATISFY THIS RELATION ARE CALLED CONFORMAL BOUNDARY STATES.

$$(\alpha_n^\mu \pm \tilde{\alpha}_{-n}^\mu)\, |B, {}^N_D\rangle\rangle = 0 .\qquad \left.\begin{array}{ll} \mu = 0 \dots p & N \\ \mu = p+1 \dots (D-1) & D \end{array}\right\} \; (*)$$

SOLUTION:

STATE THAT SOLVES $(*)$: $|Dp\rangle\rangle$

$$\boxed{|Dp\rangle\rangle} = \exp\left(- \sum_{n=1}^{\infty} \frac{1}{n} \alpha_{-n}^\mu \epsilon_{\mu\nu} \tilde{\alpha}_{-n}^\nu \right) |0\rangle\rangle$$

$|0\rangle\rangle$: FOCK SPACE GROUND STATE

$$\epsilon_{\mu\nu} = \eta_{\mu\nu} \cdot \begin{cases} + & \text{NEUMANN} \\ - & \text{DIRICHLET} \end{cases}$$

PROOF:

$k > 0$: $\alpha_k^\nu \; \exp^{-\frac{1}{k} \alpha_{-k}^\nu \epsilon_{\nu\nu} \tilde{\alpha}_{-k}^\nu} |0\rangle\rangle = (**)$

not summed

CAMPBELL BAKER HAUSDORFF FORMULA:

$$e^{sX}\, Y\, e^{-sX} = Y + s[X, Y] \quad \text{if } [X, Y] \text{ IS CENTRAL}$$

$$(**) = \Big(e^{-\frac{1}{k}\alpha_{-k}^{\nu}\epsilon_{\nu\nu}\alpha_{-k}^{\nu}}\, \alpha_k^{\nu} \; + - \frac{\epsilon^{\nu\nu}}{k}\tilde{\alpha}_{-k}^{\nu}\,\underbrace{[\alpha_{+k}^{\nu},\alpha_{-k}^{\nu}]}_{=1}\, e^{-\frac{1}{k}\alpha_{-k}^{\nu}\epsilon_{\nu\nu}\alpha_k^{\nu}} \Big) |0\rangle\rangle$$

annihilates $|0\rangle\rangle$.

$$= -\epsilon^{\nu\nu}\,\tilde{\alpha}_{-k}^{\nu}\, e^{-\frac{1}{k}\alpha_{-k}^{\nu}\epsilon_{\nu\nu}\alpha_k^{\nu}} |0\rangle\rangle.$$

$\uparrow$

± 1 DEPENDING ON N OR D.

$$|D_p\rangle\rangle = \exp\Big(-\sum \frac{1}{n}\alpha_{-n}^{\mu}\epsilon_{\mu\nu}\tilde{\alpha}_{-n}^{\nu}\Big)|0\rangle\rangle$$

SOLVES N ALONG $\mu = 0 \dots p$ & D ALONG $\mu = p+1 \dots D-1$

FOR A GENERAL 2d CFT THE COMPLETE SET OF B.C. IS HARD TO COMPUTE. BUT FOR SUFFICIENTLY NICE ONES (RATIONAL CFTS — FINITELY MANY CF. BLOCKS) THEY ARE CONSTRUCTABLE USING SO-CALLED ISHIBASHI STATES.

IN A CFT ONE WOULD ALWAYS START IMPOSING THE CF BOUNDARY CONDITION: $(L_n - \tilde{L}_{-n})|B\rangle\rangle = 0.$

WE WILL CONSTRUCT THE BOUNDARY STATES FOR THE SUPERSTRINGS: PRESERVE THE SUPER-VIRASORO.

START W/ B.C. FERMIONS.

$$\left(b_r^{\mu} + i\underset{\substack{\uparrow \\ \eta \,\hat{=}\, (-1)^F \text{ INSERTION.}}}{\eta}\tilde{b}_{-r}^{\mu}\right)|B, N, \eta\rangle\rangle = 0 \quad \text{NEUMANN.}$$

$$\left(b_r^{\mu} - i\eta\tilde{b}_{-r}^{\mu}\right)|B, D, \eta\rangle\rangle = 0 \quad \text{DIRICHLET.}$$

$$\Rightarrow \left(G_r - i\eta\tilde{G}_{-r}\right)|B, {}^{N}_{D}, \eta\rangle\rangle = 0$$

$$\left(L_n - \tilde{L}_{-n}\right)|B, {}^{N}_{D}, \eta\rangle\rangle = 0$$

PRESERVE ONE SUPERCURRENT & VIRASORO.

SOLUTION IS EASY TO CHECK FOR NS-SECTOR:

$$\boxed{|B, {}^{N}_{D}, \eta\rangle\rangle_{NS} = \exp\Big(i\eta\sum_{r>0}\underset{\substack{\uparrow \\ +\ N \\ -\ D.}}{\epsilon_{ij}}\, b^i_{-r}\tilde{b}^i_{-r}\Big)|0\rangle\rangle_{NS}}$$

NS GROUND STATE.

R-SECTOR:

FIRST WE NEED TO CONSTRUCT THE GROUND STATE THAT SATISFIES THE B.C:

RECAP: RAISING / LOWERING OPS. TO CONSTRUCT THE SPINOR REP: SO(8)

$$\left(b_0^a \pm i b_0^{a+4}\right) = \beta^{a\pm} \qquad a = 1, \ldots, 4$$

$$\left(\tilde{b}_0^a \pm i \tilde{b}_0^{a+4}\right) = \tilde{\beta}^{a\pm}$$

$$\Rightarrow |R\rangle\rangle = \prod_{a=1}^{4} \left(\beta^{a+} + i\eta \tilde{\beta}^{a+}\right) |0\rangle\rangle_R$$

SATISFIES B.C.

R GROUND STATE. $|0\rangle_R \otimes |0\rangle_R$.

$$\beta^{i-} |0\rangle_R = 0$$

$$\tilde{\beta}^{i-} |0\rangle_R = 0.$$

$$\boxed{|B, {}_{D}^{N}, \eta\rangle\rangle_R = \exp\left(i\eta \sum_{n>0} b_n^i \epsilon_{ii} \tilde{b}_n^i\right) |R\rangle\rangle}$$

i = transverse oscillators in l.c.

D_p- BRANE BOUNDARY STATES FOR IIA & IIB

$\leadsto$ GSO PROJ.

$$\begin{cases} \left(\alpha_n^i + \epsilon_i \tilde{\alpha}_{-n}^i\right) |D_p, \eta\rangle\rangle = 0 \\ \left(b_r^i + i\eta \epsilon_i \tilde{b}_{-r}^i\right) |D_p, \eta\rangle\rangle = 0 \end{cases}$$

$\epsilon_i = +$ N

$\quad\;\; -$ D.

N: $0 \ldots p$

D: $p+1 \ldots 9$

$$|D_p, \eta\rangle\rangle = \exp\left(-\sum_{n>0} \frac{1}{n} \alpha_{-n}^i \epsilon_i \tilde{\alpha}_{-n}^i - i\eta \sum_{r>0} b_{-r}^i \epsilon_i \tilde{b}_{-r}^i\right) \times$$

$$\times |\Omega\rangle \otimes |0\rangle_{NS}$$

OR $|R\rangle\rangle$.

BOSONIC VACUUM.

NEXT WE NEED TO IMPOSE THE GSO PROJECTION.

NS-SECTOR:

$$(-1)^F = (-1)^{\sum_{r>0} b_{-r}\cdot b_r - 1}$$

$$(-1)^F \, |Dp, \eta\rangle\rangle_{NS} = - \, |Dp, -\eta\rangle\rangle_{NS}.$$

$$\exp(\Sigma \eta b \tilde{b}) = \Pi (1 + \eta b \tilde{b}).$$

FERMIONIC OSC.

$$(-1)^{\tilde{F}} \, |Dp, \eta\rangle\rangle_{NS} = - \, |Dp, -\eta\rangle\rangle_{NS}.$$

$\Rightarrow$ GSO-INVARIANT STATE: $(-1)^F = 1$ IN NS SECTOR.

$$\boxed{|Dp\rangle\rangle_{NS} = \frac{1}{\mathcal{N}_{NS}} \left(|Dp, \eta=1\rangle\rangle_{NS} - |Dp, \eta=-1\rangle\rangle_{NS} \right)}$$

normalization.

R – SECTOR

$$\circledast \quad b_\pm^i = \frac{1}{\sqrt{2}}\left(b_0^i \pm i\tilde{b}_0^i\right) \qquad i = 1 \dots 8$$

$$\{b_+^i, b_-^j\} = \delta^{ij}$$

$$\{b_+ \, b_+\} = \{b_-, b_-\} = 0.$$

ZERO MODE GLUING/BOUNDARY CONDITION:

$$b_\eta^i \, |D_p, \eta\rangle\rangle_R = 0 \qquad i = 2 \dots p \qquad N$$

$$b_{-\eta}^i \, |D_p, \eta\rangle\rangle_R = 0 \qquad i = p+1 \dots 9 \qquad D$$

DEFINE: $\circledast\!\circledast$ $$|D_p, \eta = \pm\rangle\rangle_R = \prod_{i=2}^{p} b_\pm^i \prod_{i=p+1}^{9} b_\mp^i \, |D_p, \eta = \mp 1\rangle\rangle_R$$

$$b_+^i \, (RHS) = 0 \qquad i = 2 \dots p$$

SATISFIES THE GLUING.

$$b_-^i \, (RHS) = 0 \qquad i = p+1 \dots 9.$$

$\Rightarrow$ SATISFIES THE GLUING ON ZERO MODES.

GSO: $(-1)^F$ ACTION.

RECALL: $(-1)^F$ IN THE R-SECTOR:

$$(-1)^F = 16 \underbrace{\prod_{i=2}^{9} b_0^i}_{\text{ZERO MODES.}} \underbrace{(-1)^{\sum_{r>0} b_{-r}\cdot b_r}}_{\text{NONZERO MODES.}}$$

$$= \prod_{i=2}^{9} (b_+^i + b_-^i) \; (-1)^{\sum_{r>0} b_{-r}\cdot b_r}$$

$b_\pm^i$ FROM ABOVE

$$\Rightarrow \quad (-1)^F \, |D_{p}, +\rangle\rangle_R = \prod_{i=2}^{p} b_-^i \prod_{i=p+1}^{9} b_+^i \, |D_p, +\rangle\rangle_R$$

$b_+^i |\rangle = 0$ N

$b_-^i |\rangle = 0$ D

$$= |D_p, -\rangle\rangle_R. \qquad \text{By } (**)$$

$$\Rightarrow \quad \boxed{(-1)^F \, |D_p, \eta\rangle\rangle_R = |D_p, -\eta\rangle\rangle_R}$$

$$(-1)^{\tilde F} = \prod_{i=2}^{9} (b_+^i - b_-^i)\,(-1)^{\sum \tilde b_{-r} \tilde b_r}$$

$$(-1)^{\tilde F}\,|Dp,\eta\rangle\rangle_R = \prod_{i=2}^{p} (-b_-^i) \prod_{i=p+1}^{9} b_+^i\,|Dp,\eta\rangle\rangle_R .$$

$$= (-1)^{p-1}\,|Dp,-\eta\rangle\rangle_R$$

$**$

$$\Longrightarrow (-1)^{\tilde F}\,|Dp,\eta\rangle\rangle_R = (-1)^{p-1}\,|Dp,-\eta\rangle\rangle_R .$$

GSO:

$$\boxed{\begin{array}{ll} (-1)^F = 1 & \\ (-1)^{\tilde F} = \pm 1 & \begin{array}{l}\text{IIB}\\ \text{IIA}.\end{array} \end{array}}$$

TAKE LINEAR COMB. THAT ARE GSO-INVARIANT:

GSO – INVARIANT STATE:

$$|Dp\rangle\rangle_R = |Dp, \eta = 1\rangle\rangle_R + |Dp, \eta = -1\rangle\rangle_R .$$

p odd: $\quad (-1)^{\tilde{F}} = 1 \quad \Rightarrow \quad$ IIB GSO PROJECTION.

p even: $\quad (-1)^{\tilde{F}} = -1 \quad \Rightarrow \quad$ IIA " ".

$\Rightarrow$ IIB THE GSO INV. BOUNDARY STATES ARE $D(2k+1)$

IIA $\quad D(2k)$.

$\Rightarrow$ THIS IS PRECISELY CONSISTENT W/ THE RR-FORM FIELDS: IIB $\quad C_0, C_2, C_4^+$!

IIA $\quad C_1, C_3$.