STRING THEORY II

LECTURE XI

OXFORD UNIVERSITY

MMATH PHYS TT 2020

THE HETEROTIC STRING

HYBRID:

LEFT: BOSONIC STRING (26d) $X^M(z) \xrightarrow{T^{16}}$ 10d:

COMPACTIFIED ON T^{16}.

$$\left\{\begin{array}{ll} X_L^{\mu}(z) & \mu = 0 \ldots 9 \\ \boxed{X^{m}(z)} & m = 10 \ldots 25 \end{array}\right.$$

THERE IS A LOT OF PHYSICS $\rightsquigarrow$ LAST LECTURE.

RIGHT: RNS-STRING: $X_R^{\mu}(\bar{z})$, $\tilde{\psi}^{\mu}(\bar{z})$ $\mu = 0 \ldots 9$.

W/ GSO PROJECTION.

$$\Rightarrow \mathcal{H}_{Het} = \mathcal{H}_{Bosonic} \otimes \mathcal{H}_{RNS}.$$

WHY THIS IS WELLDEFINED: PS2 GHOSTS $\beta\gamma$, bc, OPE.

b, c: anticommuting
$\beta\gamma$: commuting.
} used to fix reparametrization invariance.

PS2: (PARAMETER λ): CENTRAL CHARGE:

$$c_{bc}^{\lambda=2} = -26 \qquad c_{\underset{\lambda=3/2}{\beta\gamma}\,\underset{\lambda=2}{bc}} = -26 + 11 = -15.$$

IF WE JUST CONSIDER: 10 d X^μ_L, $\tilde{X}^\mu_R$, ψ^μ CENTRAL CHARGE CONTRIBUTIONS:

$$(c_L, c_R) = (\underset{X^\mu_L}{10}, \underset{\tilde{X}^\mu_R}{10} + \underset{\psi^\mu}{5})$$

THIS IS TO BE CONTRASTED W/ THE GHOST CONTRIBUTION

$$(\underset{bc}{c_L^{ghost}}, \underset{bc,\beta\gamma}{c_R^{ghost}}) = (-26, -15)$$

$$\longrightarrow (-16, 0)$$

THIS OFFSET IN CENTRAL CHARGE OF LEFT MOVERS NEEDS TO BE CANCELLED BY EXTRA MATTER

$$\rightsquigarrow \boxed{\delta c_L = +16.}$$

THIS CAN EITER BE $\boxed{\text{16 FREE BOSONS } X^\mu(z) \quad \mu = 10 \ldots 25.}$

OR 32 FREE FERMIONS λ^I $I = 1..32.$

FREE FERMION: PS3.

16 $X^M(z)$ ARE THE COMPACTIFIED DIRECTIONS OF A 26D CRITICAL BOSONIC STRING : T^{16} TORUS.

$\Rightarrow$ CHIRAL LEFT MOVING MODES.

$\Rightarrow$ l_L MOMENTUM (RECALL: CLOSED STRING MOMENTUM LATTICE / NARAIN LATTICE Γ SIGNATURE : (n, n) FOR T^n)

$\Gamma_L \equiv (16, 0)$ SIGNATURE LATTICE.

$\Rightarrow$ EUCLIDEAN LATTICE.

THE SAME CONSISTENCY CONDITIONS APPLY FOR Γ_L AS IN LECT. 10:

Γ_L = SELF-DUAL, EVEN ; NOW ALSO EUCLIDEAN! OF DIM 16.

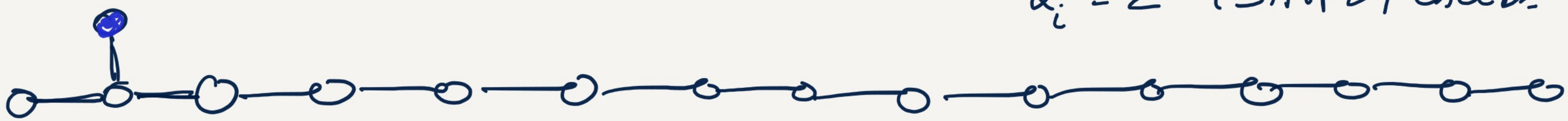
$\Rightarrow$ SUCH LATTICES ARE CLASSIFIED:

$\exists$ 2 SUCH LATTICES:

- Γ_{16} = ROOT LATTICE OF SO(32)
- $\Gamma_8 \oplus \Gamma_8$ = " " " $E_8 \oplus E_8$.

SO(32) HAS A DYNKIN DIAGRAM. (RANK 16).

$\alpha_i^2 = 2$ (SIMPLY LACED)



E_8 RANK 8 (BUT WE HAVE 2 COPIES) DYNKIN DIAG.

$\alpha_i^2 = 2$ (SIMPLY LACED).

WE CAN REPEAT THE ANALYSIS FROM THE LAST LECTURE & CONSTRUCT FROM THESE LATTICES THE CURRENT ALGEBRAS: LEVEL 1 $\hat{g}$ $\quad \hat{g} = \widehat{SO(32)}$
(LEFT MOVING).

or $\hat{g} = \hat{e}_8 \oplus \hat{e}_8$.

CENTRAL CHARGE:

$$\boxed{c_{k=1} = \frac{\dim g}{1 + g^\vee}}$$

E_8: dim 248 $\quad g^\vee = 30 \quad c = \frac{248}{31} = 8$.

$\Rightarrow E_8 \oplus E_8 \Rightarrow c = 16$ ✓

SPECTRUM	RIGHT RNS: $8_v \oplus 8_s$ GROUND STATE.	
LEFT • ∂X^μ $\mu = 0 \dots 9$	$G_{\mu\nu}$, $B_{\mu\nu}$, ϕ $\oplus$ Gravitino, Dilatino.	SAME AS TYPE I SUPERSTRING. "N=1 SUGRA".
• $\partial X^{\underset{\sim}{m}}$ $\underset{\sim}{m} = 1 \dots 16$ CARTANS OF CURRENT ALGEBRA • MOMENTUM VERTEX OP. $e^{\pm i k_L X}$ $k_L^2 = 2$	$U(1)^{16}$ GAUGE BOSONS $\oplus$ GAUGINOS. W BOSONS THAT ENHANCE THE $U(1)^{16} \longrightarrow G$ $\oplus$ " $G = E_8 \oplus E_8$ or $SO(32)$ NOTE: THE STATES ARE SPACETIME VECTORS: $\oplus$ " $\Longrightarrow$ A_μ GAUGE FIELDS FOR G GAUGE TH.	10d N=1 SUPER-YANG MILLS THEORY WITH $G = E_8 \oplus E_8$ or $SO(32)$!

THIS IS THE FIRST INSTANCE WHERE A 10D SUPERSTRING HAS CLOSED STRING GAUGE DOF !.

THIS SPECTRUM IS VERY REMINISCENT OF THE TYPE I STRING [IIB w/ Ω: WS ORIENTATION REVERSAL PROJECTED OUT].

THE SO(32) HET & TYPE I HAVE THE SAME LOW ENERGY EFFECTIVE ACTION: 10d $N=1$ SUGRA COUPLED TO $N=1$ SO(32) SYM.

TYPE I:

Recall SPECTRUM:

A_μ^a SO(32)

$G_{\mu\nu}, \Phi, C_2$ (RR) [REMEMBER: B_2 IS PROJECTED OUT BY Ω!].

+ SUSY PARTNERS.

$$S_I = \frac{1}{2\kappa_{10}^2}\int d^{10}x \sqrt{-G}\left(e^{-2\Phi}(R+4(\nabla\Phi)^2 - \frac{1}{2}|F_3|^2\right)$$

$$-\frac{1}{2g_{10}^2}\int d^{10}x\sqrt{-G}\, e^{-\Phi}\, \mathrm{Tr}\, |F_{YM}^2|$$

$$F_3 = dC_2 - \frac{\alpha'}{2}(CS(A) - CS(\omega))$$

$$CS(A) = A\wedge dA - A\wedge A\wedge A.$$

A gauge connection.
ω spin connection.
g_{10} 10d SYM gauge coupling.

FIXED BY COVARIANCE, GAUGE INV, SUSY & COMPARING W/ STRING SCATTERING THAT FIX RELATIONS BETWEEN VARIOUS PARAMETERS: $\langle V^a V^b V^c\rangle$ $\rightsquigarrow$ gauge field V.O. 3pt Fn. $\rightsquigarrow$ Coeff. of the YM Term,

FOR THE HET SO(32) STRING: SIMILAR ACTION.
PARAMETERS & FIELDS ARE RELATED AS FOLLOWS:

I	HET SO(32)
$G_I \longrightarrow$	$e^{-\Phi_{het}} G_{Het.}$
RR $F_3 \longrightarrow$	$H_3 = dB_2 - \frac{\alpha'}{4}(CS(A) - CS(\omega))$.

B_2: KALB-RAMOND B-FIELD.

F_3 COUPLES VIA C_2:

D-BRANES: $T_p \sim \frac{1}{g_{string}}$

D1 → F1 (FUNDAMENTAL STRING)

D5 → NS5 MAGNETIC DUAL NEVEAU-SCHWARZ 5-BRANE. $\left(T_{NS5} \sim \frac{1}{g_{str}^2}\right)$.

SO(32) GAUGE FIELDS. $A_\mu^a \longrightarrow A_\mu^a$

$$\boxed{\Phi_I \longrightarrow -\Phi_{HET.}}$$

"strongly coupled Type I is equivalent to weakly coupled Het".

DUALITY TYPE I & HET SO(32): STRONG-WEAK-COUPLING DUALITY !

THIS DUALITY IS QUITE DIFFERENT IN CHARACTER FROM T-DUALITY = EQUIVALENCE OF 2d CFTs.
(BOTH WEAKLY COUPLED STRINGS).
HERE: WE MAP WEAK $\leftrightarrow$ STRONG g_s-COUPLING.

TORUS-COMPACTIFICATION OF THE HET STRINGS

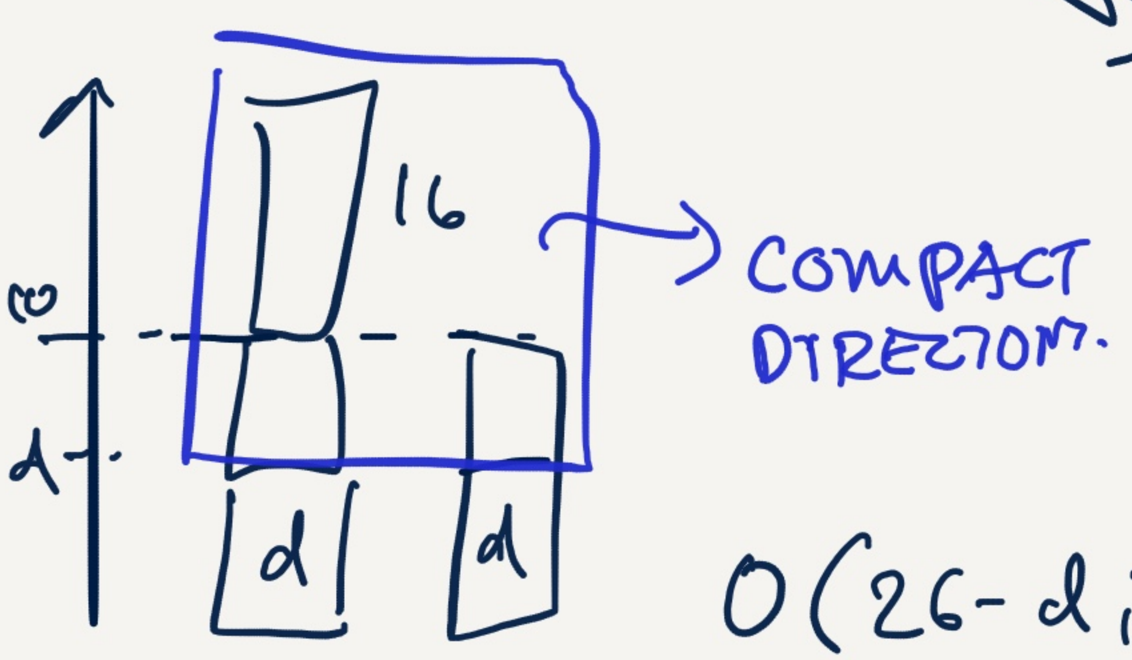
HET $E_8 \times E_8$ & $SO(32)$: ONCE WE COMPACTIFY THESE ARE IN EQUIVALENT ASWELL.

L: $X_L^M(z) \to X_L^\mu(z)$, $X_L^m(z)$

$\mu = 0 \ldots d-1$
$M = 0 \ldots 25$
$m = d \ldots 25$

R: $X_R^{M'}(\bar{z}) \to X_R^\mu(\bar{z})$, $X_R^a(\bar{z})$ $\quad a = d \ldots 9$.

COMPACTIFY HET ON A TORUS TO d dims: $(26-d, 10-d)$ SIGNATURE MOMENTUM LATTICE.



MODULI SPACE: $\mathcal{M}_d =$

$$O(26-d, 10-d, \mathbb{Z}) \backslash O(26-d, 10-d, \mathbb{R}) / O(26-d, \mathbb{R}) \times O(10-d, \mathbb{R})$$

AT A GENERIC POINT IN $\mathcal{M}_d$: THE GROUP IS

$$\begin{array}{lll} 16 & U(1) & \partial X \\ 10-d & U(1) & \text{KK modes } G_{\mu\nu} \\ 10-d & U(1) & \text{''} \qquad\qquad B_{\mu\nu} \\ \hline 36-2d & U(1)\text{s} : & G = U(1)^{36-2d}. \end{array}$$

KK = Kaluza-Klein.

16 ARE THE CARTANS OF E_8^2 or $SO(32)$.

$$\rightsquigarrow \quad E_8^2 \times U(1)^{20-2d} \quad \text{OR} \quad SO(32) \times U(1)^{20-2d}.$$

BUT THESE ARE VERY SPECIAL POINTS IN $\mathcal{M}_d$ & GENERICALLY THE GROUPS ARE DISTINCT FROM THAT.

$\Rightarrow$ $10-d$ dim $d \neq 0$ THE 2 HET STRINGS ARE JUST 2 SPECIAL POINTS IN A BIGGER MODULI SPACE OF THEORIES. $\rightsquigarrow$ CONNECTED WITHIN THIS LANDSCAPE OF THEORIES.

NEXT TIME: COMPLETE THE NETWORK OF DUALITIES OF SUPERSTRINGS: