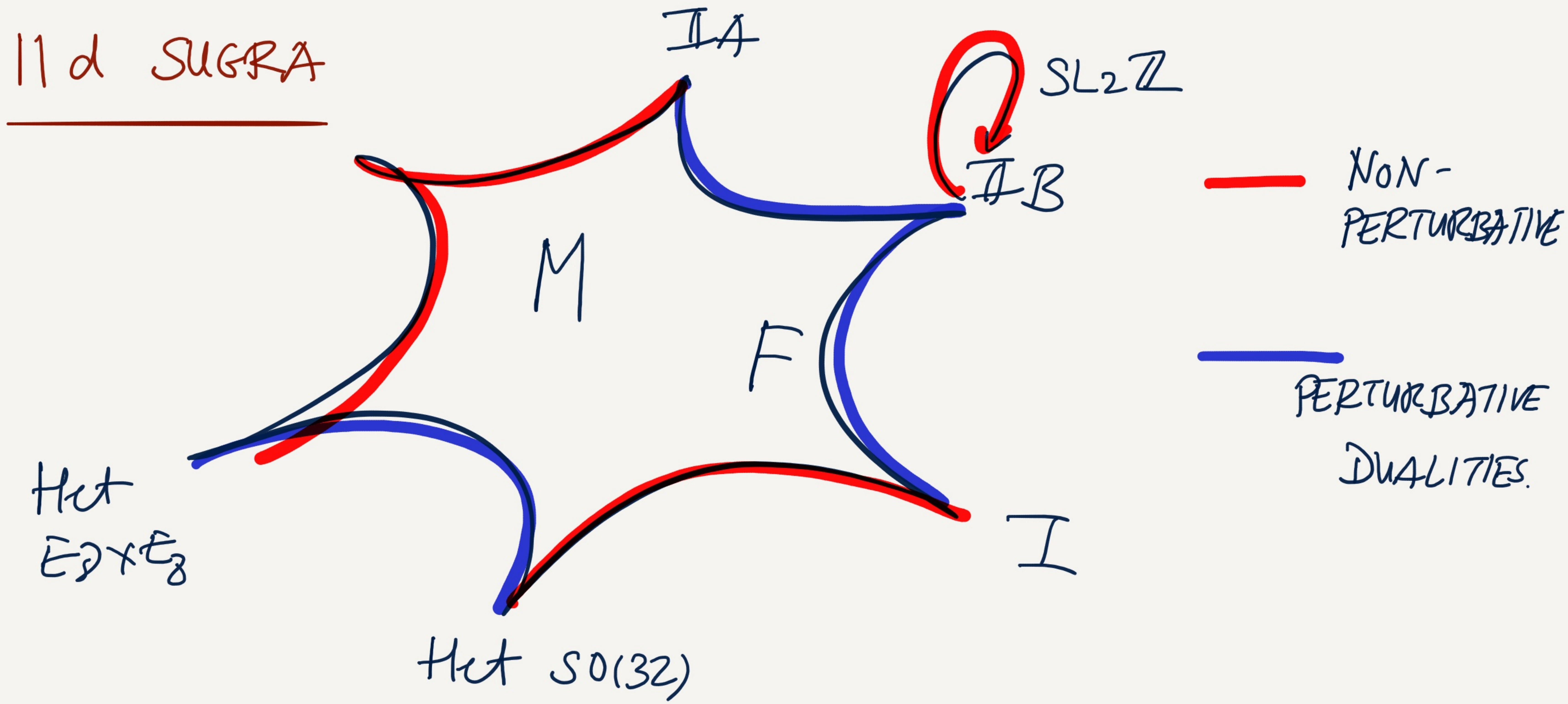
STRING THEORY II

LECTURE XII

OXFORD UNIVERSITY
MMATH PHYS TT 2020

THE PLOT THUS FAR ...



TODAY: CONNECT THE FINAL MISSING LINK — 11D SUGRA — TO IIA (& HET $E_8 \times E_8$)

ALSO WE WILL DISCUSS ANOTHER DUALITY: SELF-DUALITY OF TYPE IIB.

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11 D SUPER GRAVITY ("M-THEORY")

MAX SUSY GRAVITY THEORY IN 11 D (WHICH ITSELF IS THE MAX DIM FOR A SUGRA THEORY).

$\Rightarrow$ UNIQUE W/ THIS PROPERTY 32 SUPERCHARGES.

FROM THE REPRESENTATION TH. OF THE SUSY ALGEBRA; FIELD CONTENT OF SUGRA MULTIPLET:

$$G_{MN}, \quad C_3 \quad \text{(3-FORM)} \quad \text{w/} \quad G_4 = dC_3$$

$$+ \text{ GRAVITINO}$$

BOSONIC ACTION:

$$S_{11d} = \frac{1}{2\kappa_{11}^2} \int_{M_{11}} d^{11}x \sqrt{-G} \left[R - \frac{|G_4|^2}{2} \right] - \overbrace{\frac{1}{6} \int_{M_{11}} C_3 \wedge G_4 \wedge G_4}^{\text{CHERN-SIMONS.}}$$

C_3 COUPLES TO A 2+1 DIM. EXTENDED MEMBRANE
$\rightsquigarrow$ M2-BRANE
AND ITS MAGNETIC DUAL : 5+1 D MEMBRANE
$\rightsquigarrow$ M5-BRANE.

THESE ARE NOT D-BRANES:

- $\nexists$ OPEN STRINGS THAT END ON THEM.
  ($\because$ $\nexists$ NO STRINGS IN 11D).
- WORLDVOLUME THEORY OF N M2-BRANES OR N M5-BRANES IS NOT A SUPER-YM THEORY.

M2-BRANE: BAGGER-LAMBERT-GUSTAVSSON THEORY.

M5-BRANE: WE DO NOT KNOW WHAT THE THEORY IS EXCEPT WE BELIEVE IT IS A 6d (2,0) (MAX SUSY) SUPER-CONFORMAL FIELD THEORY.

NB: 6d MAX DIM FOR A SUPER-CONFORMAL FIELD TH. TO EXIST [NAHM].

RELATION OF 11D SUGRA TO STRING:

COMPACTIFY 11D SUGRA ON S^1 OF RADIUS R: KALUZA-KLEIN (KK) ANSATZ:

$$G^{(11)}_{MN} = \left(\begin{array}{c|c} G^{(10)}_{\mu\nu} & A_\nu \\ \hline A_\mu & e^{2\sigma} \end{array}\right)$$

A^1 = KK GAUGE FIELD

$\mu,\nu = 0 \ldots 9$

$M,N = 0 \ldots 10.$

$$(C^{(11)}_3)_{KMN} = \begin{cases} (C_3)_{\mu\nu\rho} & \text{3-FORM IN 10d.} \\ (C_3)_{10,\mu\nu} =: (B_2)_{\mu\nu} & \text{2-FORM IN 10d.} \end{cases}$$

GRAVITATIONAL COUPLINGS:

$$\kappa_{10}^2 = \frac{\kappa_{11}^2}{2\pi R}$$

INSERT THIS KK-ANSATZ INTO S_{11}:

LET: $F_4 = dC_3$ $\qquad \tilde{F}_4 = dC_3 - A_1 \wedge H_3$ $\qquad H_3 = dB_2$.

(10 d)

$F_2 = d\underline{\underline{A_1}}$ "FIELD STRENGTH OF THE KALUZA KLEIN GAUGE FIELD".

EH ACTION IN 11d

$$\rightarrow \frac{1}{2\kappa_{10}^2}\int d^{10}x\,(-G)^{1/2}\left(e^{\sigma}R - \frac{1}{2}e^{3\sigma}|F_2|^2\right) \equiv S_1$$

$|G_4|^2$ TERM IN 11d

$$\rightarrow \frac{1}{2\kappa_{10}^2}\int d^{10}x\,(-G)^{1/2}\left(e^{-\sigma}|F_3|^2 + e^{\sigma}|\tilde{F}_4|^2\right) \equiv S_2$$

CS TERM IN 11d

$$\rightarrow -\frac{1}{4\kappa_{10}^2}\int B_2 \wedge F_4 \wedge F_4 \equiv S_{CS}.$$

$G_{\mu\nu}$, σ

$B_{2\,\mu 0}$

$A_{1\mu} = C_{1\mu}$

$C_{3\,\mu\nu\rho}$

$$\Rightarrow S_{10} = S_1 + S_2 + S_{CS}.$$

Bosonic Spectrum of the TYPE IIA STRING.

IIA SUGRA ACTION IS OBTAINED BY:

$$G_{\mu\nu} \to e^{-\sigma} G_{\mu\nu} \qquad (R \to e^{\sigma} R)$$

$$\sigma = 2\,\Phi/3 \qquad \Phi \text{ IIA DILATON.}$$

GAUGE TRANSFORMATIONS:

11d: $\delta_{(2)} C_3 = d\Lambda_{(2)}$ Higher FORM GAUGE TRANSF.

$\downarrow$

10d: $\delta_{(0)} A = d\Lambda_{(0)}$

$\delta_{(0)} C_3 = \Lambda_{(0)} \wedge H_3 \qquad \delta_{(0)} \tilde{F}_4 = 0.$

11d SUGRA ON $S^1_R \longrightarrow$ IIA SUGRA

$g_s \sim R$

$g_s \gg 1$ STRONGLY COUPLED IIA $\Rightarrow$ 11d theory.
$R \gg 1.$

11d SUGRA ON S^1_R w/ $R \ll 1 \Rightarrow$ IIA 10d SUGRA

M2 – BRANE IN 11D : C_3

x^{10} not wrapped i.e. not part of the M2-worldvolume.

D2 – BRANE IN IIA $\quad T_{D_2} \sim T_{M_2} \sim \underline{\underline{\frac{1}{g_s}}}.$ ✓

x^{10} is along WV of M2

(1+1) – DIM OBJECT $(C_3)_{10,\mu\nu} = B_2$.

F1 – STRING (FUNDAMENTAL STRING)!

$$T_{F_1} = T_{M_2} \cdot R = O(g_s^0).$$

M5 – BRANE:

not wrapping x^{10}

NS5 – BRANE $\quad T_{M_5} = T_{NS5} \sim \frac{1}{g_s^2}$

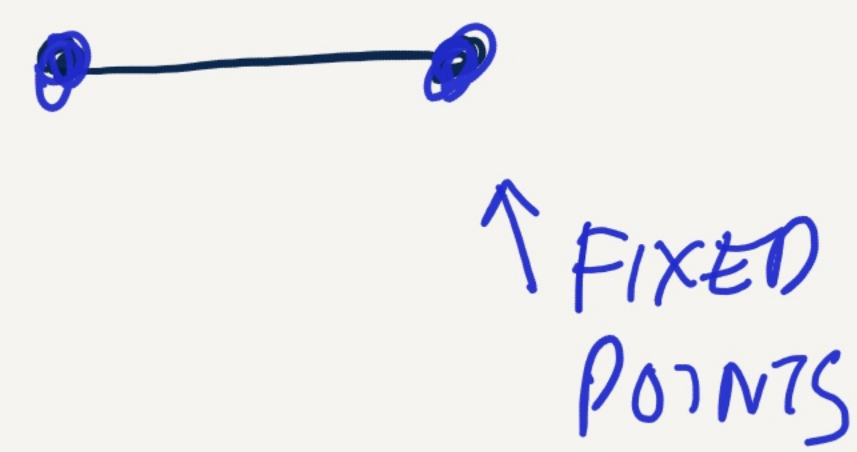
wrapping x^{10}

D4 – BRANE $\quad T_{M_5} \cdot R = T_{D_4} \sim \underline{\underline{\frac{1}{g_s}}}.$ ✓

$\Rightarrow$ 11d SUGRA SHOULD THOUGHT OF AS THE STRONGLY COUPLED IIA (10d) STRING! COUPLING IS GEOMETRIZED IN TERMS OF THE CIRCLE RADIUS R.

11d SUGRA IS ALSO RELATED TO 10d $E_8 \times E_8$ HETEROTIC STRINGS [Horava – Witten]:

$$11d \text{ SUGRA on } S^1/\mathbb{Z}_2 :$$



$= \text{INTERVAL}$

ANOMALY FROM THESE FIXED POINTS:

$\rightsquigarrow$ INTRODUCE "E_8-WALLS" AT THE ENDPOINTS.

$\rightsquigarrow$ REDUCE SIZE OF INTERVAL TO 0:

$E_8 \times E_8$ HET SUGRA ACTION.

[POLCHINSKI II].

ONE MORE DUALITY IN 10D: IIB SELF-DUALITY.

FIELD CONTENT OF IIB SUGRA:

$$G_{\mu\nu}, \quad \Phi, \quad B_2$$
$$C_0, \quad C_2, \quad C_4^{+}$$

$$F_5 = * F_5$$
$$F_1 = d C_0$$
$$F_3 = C_0\, dB_2 - dC_2 .$$

$$F_5 = dC_4 - \frac{1}{2} C_2 \wedge dB_2 + \frac{1}{2} B_2 \wedge dC_2 .$$

$\exists$ A NICE REDEFINITION OF THE FIELDS AS FOLLOWS:

$$\tau \equiv C_0 + i e^{-\Phi}$$

AXIO-DILATON

C_0 $\quad$ Φ

$$G_3 = dC_2 - \tau\, dB_2$$

$$\Rightarrow S_{IIB} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G}\left(R - \frac{\partial_\mu\tau\partial^\mu\bar{\tau}}{2(\mathrm{Im}\,\tau)^2} - \frac{1}{2}\frac{|G_3|^2}{\mathrm{Im}\,\tau} - \frac{1}{4}|\tilde{F}_5|^2\right)$$

$$+\frac{1}{8i\kappa_{10}^2}\int\frac{1}{\mathrm{Im}\,\tau}C_4\wedge G_3\wedge\bar{G}_3$$

$$\tau = \underset{\underset{C_0}{\shortparallel}}{\tau_1} + i\underset{\underset{e^{-\Phi}}{\shortparallel}}{\tau_2}$$

THIS MAKES MANIFEST THE FOLLOWING SYMMETRY:

- $\tau \longrightarrow \dfrac{a\tau+b}{c\tau+d}$ $\quad ad-bc=1$

  (CLASSICALLY: $SL_2\mathbb{R}$)
  QUANTUM: $SL_2\mathbb{Z}$.

- $\begin{pmatrix} C_2 \\ B_2 \end{pmatrix} \longrightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} C_2 \\ B_2 \end{pmatrix}$ $\quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL_2\mathbb{R}$.

PS2: INVARIANCE & $\boxed{S:\ \tau\to -1/\tau}$

$T:\ \tau\to\tau+1.$ ✓.

$\tau = C_0 + ie^{-\Phi}$

$\Rightarrow$ STRONG-WEAK COUPLING DUALITY.

$\Rightarrow$ S-DUALITY OR $SL_2\mathbb{Z}$ $(\mathbb{R})$ DUALITY OF IIB.

KEY: D3-BRANES ARE MAPPED INTO D3-BRANES HOWEVER THE DUALITY INDUCES A STRONG-WEAK-DUALITY ON THE WV THEOR: 4d MAX SYM THEORY.. ("4d N=4 SYM").

$$\tau^{IIB} \longrightarrow \tau^{D3} = \theta + i\frac{1}{g_{YM}^2}$$

4d N=4 SYM HAS AN S-DUALITY: MONTONEN OLIVE DUALITY:

$$\begin{cases} \tau^{D3} \longrightarrow \dfrac{a\tau+b}{c\tau+d} \\ \text{GAUG GP. } G \longrightarrow {}^{L}G = \text{LANGLANDS DUAL GP.} \end{cases}$$

[KAPUSTIN & WITTEN]

OVERVIEW:

M-THEORY

11D SUGRA
M2, M5

$S^1_{R_{11}}$ → 10D IIA

$S^1/\mathbb{Z}_2$ → HET

S-DUALITY

10D IIA
$R_{11} \sim g_s$
F1, D2, D4, D6, D8
NS5

10D IIB
D1, D3, D5, D7, D9
NS5

$\xrightarrow{\Omega}$

10D I
D1, D5, D9
SO(32)
SYM

⟷

HET
SO(32)
F1, NS5
SO(32)
SYM.

HET
$E_8 \times E_8$
F1, NS5
$E_8 \times E_8$
SYM

10D IIA $\xrightarrow{S^1_A}$ 9D IIA

10D IIB $\xrightarrow{S^1_B}$ 9D IIB

$$9D \text{ IIA} \xleftrightarrow{R_A = \frac{\alpha'}{R_B}} 9D \text{ IIB}$$

HET SO(32) $\xrightarrow{S^1}$ 9d HET STRING $\xleftarrow{S^1}$ HET $E_8 \times E_8$

MANY MORE DUALITIES EXIST IN LOWER DIM.
[ASHOKE SEN '90s REVIEW].
POLCHINSKI II
E.G: $\text{IIA}/\underline{\underline{K3}} \cong \text{HET}/T^4$

STRONG COUPLING VERSION OF TYPE IIB: F-THEORY.

τ = COMPLEX MODULUS OF A TORUS.

WE NEED TO ANALYSE COMPACTIFICATIONS OF STRING TH. BEYOND TORI:

- TORI PRESERVE SUSY
  $\Rightarrow$ WE NEVER GET MIN SUSY THEORIES IN LOWER DIMS.
  (NO 4d $\mathcal{N}=1$ e.g.).

- FLUX BACKGROUNDS (NON-TRIVIAL BACKGROUND VALUES FOR RR-POTENTIALS & NSNS FORMS.

$\longrightarrow$ NEXT LECTURE.