STRING THEORY II

LECTURE XIII

OXFORD UNIVERSITY
MMATH PHYS TT 2020

SO FAR WE HAVE DISCUSSED

- 10 D SUPERSTRINGS (OR 11D SUGRA)
- T^N TORUS-COMPACTIFICATIONS.

TODAY: WHAT COMPACTIFICATION SPACES ARE CONSISTENT IN SUPERSTRING THEORY — BEYOND TORI.

$$10D \text{ SPACETIME} = \underbrace{\mathbb{R}^{1,9-d}}_{\substack{\text{Flat Minkowski} \\ \text{spacetime.}}} \times \underbrace{M_d}_{\substack{\text{COMPACT } d\text{-dim} \\ \text{space. (e.g. } T^d\text{).}}}$$

STUDY KALUZA-KLEIN REDUCTION ON M_d to 10-d THEORY.

$\boxed{\text{KEY}}$ WHY IS THERE SUCH A DEEP CONNECTION BETWEEN STRING THEORY & GEOMETRY?
"GEOMETRY OF M_d DICTATES THE EFFECTIVE FIELD TH. (EFT) IN 10-d DIM !".

E.G. T^d WE HAVE SEEN ENHANCED GAUGE SYMMETRIES FROM SPECIFIC CHOICE OF PARAMETERS OF THE TORUS.

INPUT PHYSICAL CONDITIONS ON THE EFT.

E.G. 4 DIM COMPACTIFICATIONS: $d=6$ SIX DIM COMPACT SPACE: TO MODEL THE UNIVERSE WE WANT

q
- 4d GRAVITY
- 4d GAUGE THEORY WITH $G = SU(3) \times SU(2) \times U(1)$
  + MATTER (FERMIONS & SCALARS)
  + INTERACTIONS (YUKAWA COUPLINGS).

TO HAVE CONTROL OVER QUATUM CORRECTIONS WE ALWAYS IMPOSE THE THEORY IN $\mathbb{R}^{1,9-d}$ TO BE AT LEAST MINIMALLY SUPERSYMMETRIC. $\rightsquigarrow$ CONTROLS α' CORRECTIONS.

$\rightsquigarrow$ TRUST THE EFT & KK-REDUCTION.

4d: $N=1$ SUSY.

6d: $N=(1,0)$ SUSY.

5d: $N=1$ "

NOTE: ONE CAN STUDY ~~SUSY~~ IN STRING THEORY ONCE ONE HAS ESTABLISHED A SUSY VACUUM.

$\rightsquigarrow$ NON-PERTURBATIVE EFFECTS THAT ARE USED TO ~~SUSY~~

$\rightsquigarrow$ COMPUTE SOFT SUSY BREAKING

EG. SUSY STANDARD MODEL + ~~SUSY~~ SECTOR & STUDY THE STANDARD SUSY-BREAKING MEDIATIONS GRAVITY, MEDIATON OR GAUGE MEDIATION.
ANOMALY

GOAL: FIRST DETERMINE $N=1$ SUSY COMPACTIFICATIONS.

$\Rightarrow$ WHAT CONDITIONS DOES THIS PRESERVATION OF $N=1$ (MIN) SUSY IN $10-d$ DIM IMPOSE ON M_d?

BASIC CONDITION:

1. SUGRA NEEDS TO BE WELL-DEF ON M_d: Spin manifold.

2. $d=6$: KK-REDUCTION

$$G_{MN}(x,y) = \left(\begin{array}{c|c} g_{\mu\nu}(x) & \\ \hline & g_{mn}(y) \end{array} \right)$$

$g_{\mu\nu} = \eta_{\mu\nu}$ METRIC ON $\mathbb{R}^{1,3}$

g_{mn} METRIC ON M_6 w/ COORDINATES y.

$$SO(1,9) \rightarrow SO(1,3) \times SO(6)$$

LIKEWISE WE DECOMPOSE THE SPINORS:

$$SO(1,9) \longrightarrow SO(1,3) \times SO(6)$$

$$\underbrace{\epsilon = \underline{16}}_{\text{(Majorana Weyl)}} \longmapsto \boxed{\left((\underline{2},\underline{1}) ; \underline{\bar{4}} \right) \oplus \left((\underline{1},\underline{2}) ; \underline{4} \right)}$$

$SO(1,3) \cong SU(2) \times SU(2)$

WEYL SPINOR: $(\underline{2},\underline{1})$

CONJUGATE WEYL SPINOR: $(\underline{1},\underline{2})$

$\boxed{}$:

(2,1) & (1,2) AS THE 4d WEYL / CONJ WEYL SPINORS.

$\underline{\bar{4}}, \underline{4} \hat{=}$ MULTIPLICITIES.

$SO(6) \cong SU(4)$ ROTATION GP IN 6d (EUCLIDEAN)

$\left.\begin{array}{l} \underline{6} \text{ VECTOR} \\ \underline{4}, \underline{\bar{4}} \text{ SPINOR} \end{array}\right\}$ REPS.

IF $M_6 = T^6$: PRESERVE ALL OF THESE SPINOR COMPONENTS IN 4d.

$\Rightarrow$ MAX SUSY TH. IN 4D.

(T^6 IS FLAT). IN GENERAL M_6 IS CURVED & NOT ALL SPINOR COMPONENTS WILL BE PRESERVED.

SUSY BACKGROUNDS FOR STRING COMPACTIFICATIONS

$\epsilon_\alpha \hat{=}$ SUSY TRANSFORMATION PARAMETERS IN 10D.

$Q^\alpha \hat{=}$ SUPERCHARGES.

SUSY VACUUM: $\epsilon \cdot Q \,|\,\text{VACUUM}\rangle = 0.$

IF WE HAVE NONTRIVIAL "BACKGROUND FIELDS":

E.G. METRIC, B-FIELD, F_p-FORM etc: Φ

SUSY BACKGROUND: $\langle \delta_\epsilon \Phi \rangle_{\text{VACUUM}} = 0.$

$\delta_\epsilon \Phi_{\text{BOSONIC}} \sim \Phi_{\text{FERMIONS}} \equiv 0.$ (NO FERMION CONDENSATES IN BACKGROUND)

$\Rightarrow \quad \langle \Phi_{\text{FEMIONS}} \rangle_{\text{VACUUM}} = 0.$

$\delta_\epsilon \Phi_{\text{FEMIONS}}$ SHOULD ALSO VANISH: THIS IS A COMBINATION OF Φ_{BOSONIC}:

$$\boxed{\langle \delta_{\underline{\epsilon}} \Phi_{\text{FERMIONS}} \rangle = 0}$$

SUGRA:

Φ_{FERMION}: GRAVITINO Ψ_μ

DILATINO λ.

SUSY-VARIATIONS : GRAVITINO Ψ_M :

Dilaton

$$\circledast \quad \boxed{\delta_\epsilon \Psi_M = \nabla_M \epsilon + \cancel{H}_M \epsilon + e^{\Phi} \sum_n \cancel{F}_n \Gamma_M \epsilon \overset{!}{=} 0}$$

$$\nabla_M = \partial_M + \frac{1}{2} \omega_M{}^{ab} \Gamma_{ab}$$

$\Gamma_a \hat{=}$ 10d Γ MATRICES.

$\Gamma_{ab} = \frac{1}{2} [\Gamma_a, \Gamma_b]$.

$$\cancel{H}_M = H_{M_1 M_2 M_3} \Gamma^{M_1 M_2 M_3}$$

$\cancel{F}_n$ F_n depending on IIB OR IIA BEING ODD OR EVEN.

ω is THE SPIN CONNECTION : CONNECTION ON THE SPIN BUNDLE OVER 10 dim SPACETIME.

$$\omega_M{}^{ab} = \frac{1}{2} \left(\Omega_{MNR} - \Omega_{NRM} + \Omega_{RMN} \right) e^{Na} e^{Rb}$$

e = 10-BEIN.

$$\Omega_{MNR} = \left(\partial_M e_N{}^a - \partial_N e_M{}^a \right) e_{aR}.$$

$\circledast$ SOLVE THIS COND. FOR SUSY VACUUM CONFIGURATIONS OF e_N^a (i.e. METRIC), H, F_n, ϕ & ϵ !

WHAT SOLUTIONS EXIST OF ⊛ W/ NONTRIVIAL ϵ (SOME COMPONENTS OF THE SPINOR).

- $\phi = H = F_n = 0$ $\quad$ $\omega = 0$ (FLAT METRIC) $\simeq T^6$

  $\Rightarrow \quad \partial_M \epsilon = 0 \Rightarrow$ CONSTANT SPINORS.

  \# SUSY PRESERVED IS SIMPLY THE # SUSY AS IN 10 d (TYPE I : ONE ϵ $\quad$ 4d $N=4$

  II : $\quad 2\epsilon$ $\quad$ 4d $N=8$).

- $H = F_n = 0 = \phi$: $\quad \omega \neq 0$

$$\delta_\epsilon \psi_M = \boxed{\nabla_M \epsilon = 0}$$

THIS EQ. IS SOMETIMES REFERED TO AS THE KILLING SPINOR EQ.

$\rightsquigarrow$ METRIC & ϵ.

$\boxed{\text{PS4}}$ $\quad [\nabla_M, \nabla_N] = \frac{i}{2} R_{MN}{}^{ab}\, \Gamma_{ab}$.

$$\nabla_M \epsilon = 0 \quad \Rightarrow \quad 0 = [\nabla_M, \nabla_N]\epsilon = \underbrace{\tfrac{i}{2} R_{MN}{}^{ab}\,\Gamma_{ab}\,\epsilon \overset{!}{=} 0}_{(**)}$$

NOTE 1: Γ_{ab} THIS GENERATES A SUBALGEBRA
($R^{ab}\Gamma_{ab}\epsilon = 0$ IS NOT $\Gamma_a \epsilon = 0$).

Γ_{ab} GENERATES A SUBGROUP OF

$SO(6) \supset \underline{\underline{SU(3)}}$ (SPECIAL UNITARY GP).

↑ LOCAL LORENTZ GR. ON M_6.

NOTE 2: WE IMPOSE THIS CONDITION ONLY ALONG THE COMPACT DIRECTIONS $M_d = M_6$
SO WE RETAIN COMPLETE SPINOR REP. FOR THE $SO(1,3)$ IN $\mathbb{R}^{1,3}$.

$(**)$ $$\boxed{R_{mn} = 0}$$

RICCI TENSOR FOR M_6 HAS TO VANISH.

m, n = coord. along M_6.

$$SO(1,9) \longrightarrow SO(1,3) \times SO(6)$$

$$\epsilon \hat{=} 16 \longrightarrow ((2,1), \cancelto{1}{\bar{4}}) \oplus ((1,2), \cancelto{1}{4}).$$

$M \rightarrow \mu : \mathbb{R}^{1,3}$: $\omega_\mu = 0$ CONSTANT SPINORS ALONG $\mathbb{R}^{1,3}$.

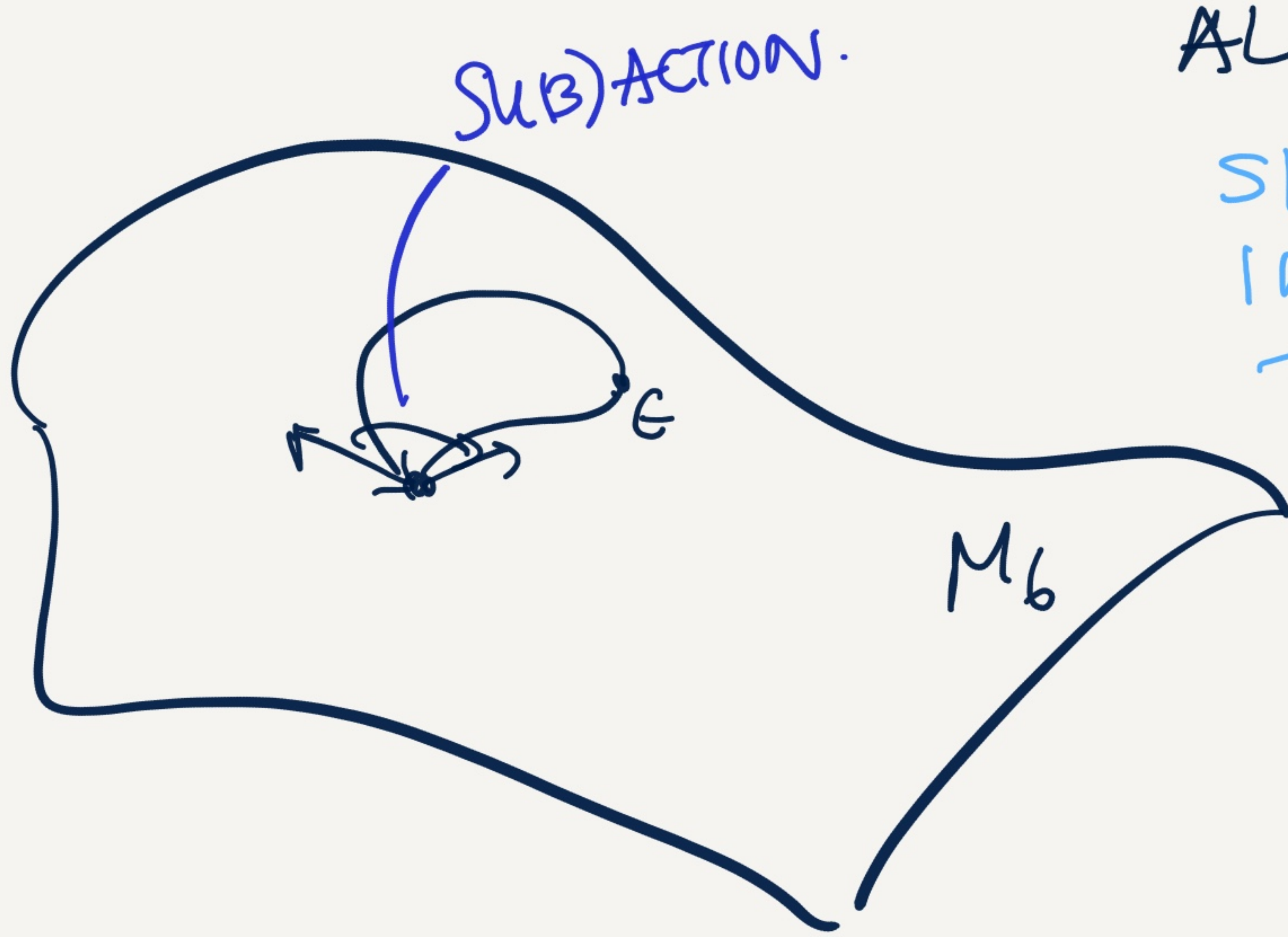
$\searrow m$: $\nabla_m \epsilon = 0$.

$$SU(4) \cong SO(6) \longrightarrow SU(3)$$

BY THE ACTION IN (**).

$$4 \longrightarrow 1 \oplus 3$$

INVARIANT SPINOR COMPONENT ALONG M_6.



SINGLET COMPONENT IS LEFT INVARIANT UNDER THE PARALLEL TRANSPORT.

$$\epsilon \cong 16 \longrightarrow (2,1) \otimes 1 \oplus (1,2) \otimes 1.$$

$\hat{=}$ 4d $N=1$ SUSY.

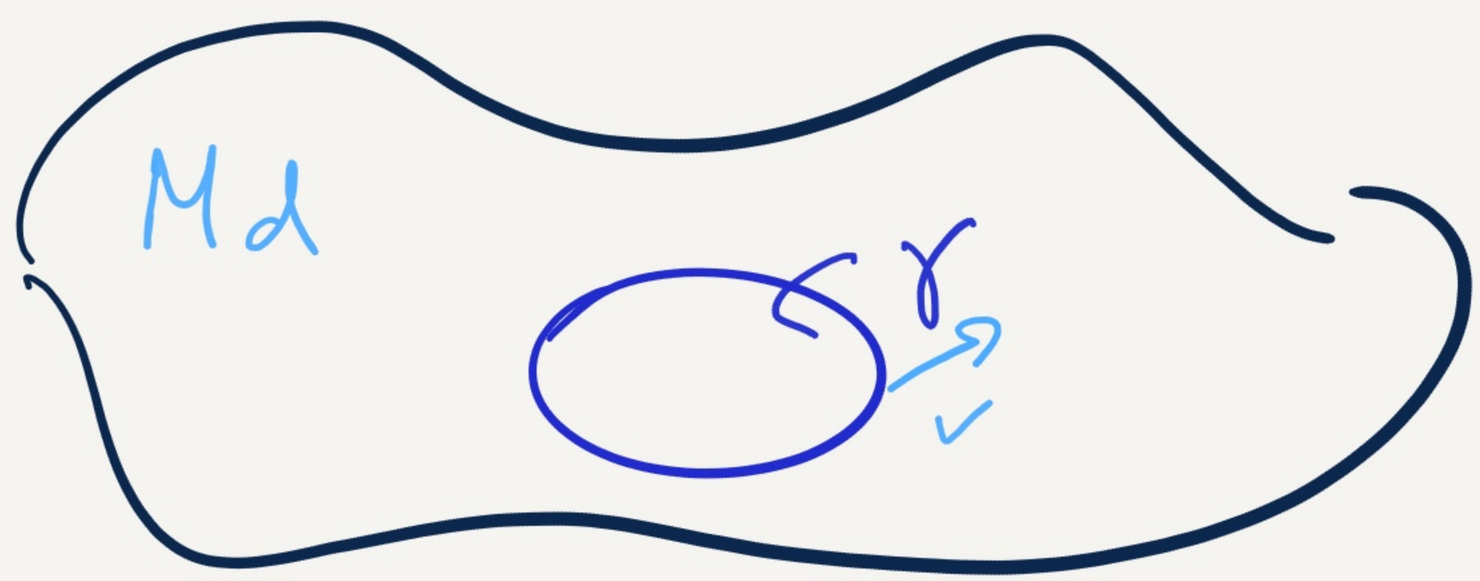
TYPE I ON M_6 w/ $R_{mn}=0$: 4d $\mathcal{N}=1$

II 4d $\mathcal{N}=2$.

$\Rightarrow$ SPACES w/ THIS PROPERTY $R_{mn}=0$ ARE CALLED RICCI FLAT.

$\boxed{M_6}$: CALABI-YAU MANIFOLDS.

HOLONOMY :



γ = closed loop.

PARALLEL TRANSPORT ALONG γ: FOR A GENERAL M_d :

$$v \longrightarrow Uv \qquad U \in SO(d)$$

SPACES WHERE $U \in G \subsetneq SO(d)$: REDUCED HOLONOMY

$d=6$: $SU(3) \subset SO(d)$ $\quad R_{mn} = 0 \quad$ CALABI-YAU 3-MANIFOLDS

M_{2d} : $SO(2d)$ REDUCED TO $SU(d)$ HOLONOMY

CALABI YAU d- MANIFOLDS.

(COMPLEX d DIM MANIFOLDS).

M_7 : $G = G_2 \subsetneq SO(7)$

M_8 : $G = \mathrm{Spin}(7) \subsetneq SO(8)$

EXCEPTIONAL HOLONOMY MANIFOLDS.

MATHEMATICALLY VERY ACTIVELY RESEARCHED.

FOR HETEROTIC STRING: GAUGINO VARIATION

$$\delta\lambda = F_{mn}\,\Gamma^{mn}\,\epsilon \overset{!}{=} 0$$

BACKGROUND FOR GAUGEFIELD

$\Rightarrow$ DETERMINES GAUGE-BUNDLE FOR THE COMPACTIFIC.
(GEOMETRY + ").