STRING THEORY II

LECTURE XV

OXFORD UNIVERSITY
MMATH PHYS TT 2020

SUPERSYMMETRIC COMPACTIFICATIONS (W/O FLUXES) $\hookrightarrow$ [BECKER[2], SCHWARZ] CUP. "STRING TH. & M-TH".

$\Rightarrow$ M_d THE COMPACTIFICATION SPACE HAD TO ADMIT SPINORS:

$\nabla_M \epsilon = 0 \Rightarrow$ RICCI-FLAT.

II, I, HET W/ $M_{d=6}$: 4d SUSY COMPACTIFICATIONS W/ M_6 = CALABI-YAU MANIFOLD (COMPLEX, KÄHLER, RICCI-FLAT, SU(3)-REDUCED / HOLONOMY).

$h^{p,q}$ HODGE # : CY 3-FOLD :

$$
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & h^{1,1} & & 0 & \\
1 & & h^{1,2} & & h^{1,2} & & 1 \\
 & 0 & & h^{1,1} & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 1 & & &
\end{array}
$$

$(h^{1,1}, h^{1,2})$

TECHNICALLY T^6 IS NOT REALLY A CY, HOLONOMY IS TRIVIAL.

(FORMALLY STILL APPLY ALL OF THE STRUCTURES: $T^2_{\tau_1} \times T^2_{\tau_2} \times T^3_{\tau_3}$)

$\rightarrow$ SUSY IN 4D $N > 1$.

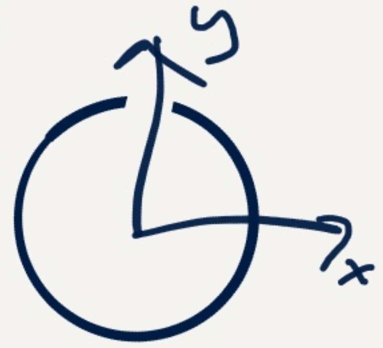
ANOTHER CLASS OF EXAMPLES: HYPER SURFACES IN PROJECTIVE SPACE $= \mathbb{P}^n = \{ (\underbrace{z_0 \dots z_{n+1}}_{\underline{z}}) \in \mathbb{C}^{n+1} : \underline{z} \sim \underline{w}$ if: $\exists \lambda \neq 0 : \underline{z} = \lambda \underline{w} \}$.

HYPERSURFACE: $P_d(\underline{z}) = 0 \quad \subseteq \mathbb{P}^n$.

$$P_d(\lambda \underline{z}) = \lambda^d P_d(\underline{z}).$$

"HOMOGENEOUS POLYNOMIAL OF DEGREE d".

$x^2+y^2=1$



THEN:

$$X_d = \{ [z_0, \dots, z_n] \in \mathbb{P}^n : P_d(z) = 0 \}.$$

DEFINE CALABI-YAU d-FOLDS ($\dim_{\mathbb{C}} X_d = d$).

$n=2$: $d=3$: TORUS.

$n=3$: $d=4$: CY ~~2-FOLD~~ (KUMMER-SURFACE)

K3-SURFACES.

$n=4$: $d=5$: CY3: QUINTIC IN $\mathbb{P}^4$: $z_1^5 + z_2^5 + z_3^5 + z_4^5 + z_5^5 = 0$

$\underline{z} \in \mathbb{P}^4$.

REMARK FOR MATH BACKGROUND STUDENTS.

CY CONDITION OF RICCI-FLATNESS IN ALGEBRAIC-GEOMETRIC TERMS MEANS:

K_M (CANONICAL CLASS OF M) IS TRIVIAL.

$= \mathcal{O}_M$. $(c_1(M) = 0)$.

FOR A HYPER SURFACE X_d IN $\mathbb{P}^n$:

$$[c_1(X_d)] = \underbrace{(n+1-d)}_{\overset{!}{=}0.} [H]$$

HYPERPLANE CLASS OF $\mathbb{P}^n$.

$$\Rightarrow (n,d) = (3,4), (4,5), \ldots$$
$$(2,3)$$

WHAT ARE $h^{1,1}$ & $h^{1,2}$ OF THE QUINTIC:

- $h^{1,1}$: KÄHLER FORM INDUCED FROM THE AMBIENT $\mathbb{P}^4$ (ω). $h^{1,1} = 1$.

- $h^{1,2}$: SHAPE PARAMETERS i.e. COMPLEX STRUCTURE DEFORMATIONS. FOR THE HYPERSURFACES THESE ARE COMPUTED BY COUNTING THE POLYNOMIAL DEFORMATIONS MODULO IDENTIFIC. IN $\mathbb{P}^4$ (OR THE AMBIENT SPACE).

$$\rightarrow P_5(z) + \sum \alpha_{ijklm} z_1^i z_2^j z_3^k z_4^l z_5^m$$

$$h^{1,2} = 101$$

$$\Rightarrow \quad \begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & 1 & & 0 & \\ 1 & & 101 & & 101 & & 1 \\ & 0 & & 1 & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array}.$$

NOW ON TO COMPACTIFICATIONS OF STRINGS ON CYs.

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(1) HETEROTIC $E_8 \times E_8$ ON CY3 $\longrightarrow$ 4d $\mathcal{N}=1$ SUSY GRAVITY + SYM.

$\Rightarrow$ PHENOMENOLOGICAL MOTIVATION:

GOAL: ENGINEER THE SPECTRUM OF THE MINIMALLY SUSY STANDARD MODEL OR A SUSY GRAND UNIFIED TH. (GUT) w/ $G = SU(5)$ or $SO(10)$ UNIFICATION GP.

(2) IIA, IIB ON CY3 $\longrightarrow$ 4d $\mathcal{N}=2$ SUSY.

$\Rightarrow$ MATHEMATICAL MOTIVATION.

1 10 d $\mathcal{N}=1$ SUPERGRAVITY + 10 d $\mathcal{N}=1$ SYM w/ $G = E_8 \times E_8$.

$\approx$ HET $E_8 \times E_8$ EFFECTIVE DESCRIPTION.

KILLING SPINOR EQS:

- GRAVITINO: $\delta \Psi_M = 0 \Rightarrow R_{mn} = 0$

10 d SPACETIME: $\mathbb{R}^{1,3} \times M_6$ M_6 is CY3.

- GAUGINO: $\delta \lambda = 0 \Rightarrow F_{mn} \Gamma^{mn} \epsilon = 0$.

BACKGROUND VALUE OF THE $E_8 \times E_8$ GAUGE FIELD.

DECOMPOSING THE $\delta\lambda = 0$ COND.
FOR COMPLEX & KÄHLER: w/ $R_{mn} = 0$.

$$F_{ij} = 0 \qquad F_{\bar{i}\bar{j}} = 0$$

F = 2-FORM
$\Omega^{2,0}, \Omega^{1,1}, \Omega^{0,2}$

$$\Leftrightarrow F^{(2,0)} = 0 \qquad \Leftrightarrow F^{(0,2)} = 0 \qquad \boxed{g^{i\bar{j}} F_{i\bar{j}} = 0}$$

$F_{\mu\nu}$ ALONG $\mathbb{R}^{1,3}$ HAVE TRIVIAL BACKGROUND VALUES.

HET: NSNS 2-FORM B_2

GAUGE FIELD A

spin connection.

$$\tilde{H}_3 = dB_2 - \frac{\alpha'}{4}\left(CS(A) - CS(\omega_{spin}) \right) \qquad CS = A\,dA - \frac{2}{3}A^3.$$

$$\xrightarrow{d} \quad d\tilde{H}_3 = -\frac{\alpha'}{4}\left(\mathrm{Tr}(R_2 \wedge R_2) - \mathrm{Tr}(F_2 \wedge F_2) \right).$$

FOR $d\tilde{H}_3 = 0$: NO BACKGROUND FLUX:

$$\boxed{\mathrm{Tr}\, R_2 \wedge R_2 = \mathrm{Tr}\, F_2 \wedge F_2.} \quad (*).$$

$\Rightarrow$ THE BACKGROUND GAUGE FIELD CANNOT BE SIMPLY SET TO ZERO.

$(*)$ HAS A SIMPLE SOLUTION: STANDARD EMBEDDING OF SPIN CONNECTION INTO GAUGE CONNECTION!

$$A = \omega_{spin}. \qquad \omega_{spin} = \text{SPIN CONNECTION OF } M_6.$$

FOR: CY_3 : ω_{spin} IS CONNECTION IN $SU(3)$ (REDUCED HOLONOMY OF CY3)

$A = \omega_{spin}$: A SU(3) - CONNECTION:

$$E_8 \supset SU(3) \times E_6$$

E_6 = COMMUTANT OF SU(3) IN E_8.

ADJOINT (GAUGE BOSONS)

$$\underset{\substack{\text{ADJOINT OF} \\ SU(3)}}{248 \longrightarrow (8,1)} \oplus \underset{\substack{\text{ADJOINT OF} \\ E_6}}{(1,78)} \oplus (3,27) \oplus (\bar{3},\overline{27})$$

"BIFUNDAMENTALS" OF $SU(3) \times E_6$.

$\langle F_{SU(3)} \rangle \neq 0 \Rightarrow$ HET STRING HAS "ONLY" $E_8 \times E_6$. GAUGE SYMMETRY.

See BLT.

Table 14.2 Massless spectrum of the $E_8 \times E_8$ heterotic string on CY_3 with standard embedding

Multiplets	Component fields	Multiplicity
Gravity	$g_{\mu\nu}, \bar{\psi}_{\mu\dot{\alpha}}\eta, \psi_{\mu\alpha}\eta_{\bar{\imath}\bar{\jmath}\bar{k}}$	1
Chiral	$\Phi, B_{\mu\nu}, \lambda_\alpha\eta, \bar{\lambda}_{\dot{\alpha}}\eta_{\bar{\imath}\bar{\jmath}\bar{k}}$	1
Chiral	$g_{ij}, g_{\bar{\imath}\bar{\jmath}}, \psi_\alpha\eta_{\bar{\imath},\bar{\jmath}}, \bar{\psi}_{\dot{\alpha}}\eta_{i,\bar{\jmath}\bar{k}}$	$h^{2,1}$
Chiral	$g_{i\bar{\jmath}}, B_{i\bar{\jmath}}, \psi_\alpha\eta_{i,\bar{\jmath}}, \bar{\psi}_{\dot{\alpha}}\eta_{\bar{\imath},\bar{\jmath}\bar{k}}$	$h^{1,1}$
$\boxed{\text{Vector}}$	$A_\mu^{(248)}, \bar{\chi}_{\dot{\alpha}}^{(248)}\eta, \chi_\alpha^{(248)}\eta_{\bar{\imath}\bar{\jmath}\bar{k}}$ ← VM	1 OF E8
$\boxed{\text{Vector}}$	$A_\mu^{(1,78)}, \bar{\chi}_{\dot{\alpha}}^{(1,78)}\eta, \chi_\alpha^{(1,78)}\eta_{\bar{\imath}\bar{\jmath}\bar{k}}$ ← VM	1 OF E6
SU(3) → Chiral	$(A_i^{(8,1)}, \chi_\alpha^{(8,1)}\eta_{\bar{\imath}\bar{\jmath}}) + (A_{\bar{\imath}}^{(8,1)}, \bar{\chi}_{\dot{\alpha}}^{(8,1)}\eta_{\bar{\imath}})$	$h^1(\mathrm{End}\,T)$
Chiral	$(A_{\bar{\imath}}^{(\bar{3},\overline{27})}, \chi_\alpha^{(\bar{3},\overline{27})}\eta_{\bar{\imath}}) + (A_i^{(3,27)}, \bar{\chi}_{\dot{\alpha}}^{(3,27)}\eta_{\bar{\imath}\bar{\jmath}})$	$h^{1,1}$
Chiral	$(A_{\bar{\imath}}^{(3,27)}, \chi_\alpha^{(3,27)}\eta_{\bar{\imath}}) + (A_i^{(\bar{3},\overline{27})}, \bar{\chi}_{\dot{\alpha}}^{(\bar{3},\overline{27})}\eta_{\bar{\imath}\bar{\jmath}})$	$h^{2,1}$

A=ω SU(3)

$\left.\right\}$ $E_8 \times E_6$ gauge group in 4d.

BIFU IN (3,27)

<u>$E_6 \times E_8$:</u>

- $E_6 \supset SU(5), SO(10)$. GUT.
- TURN ON ANOTER $F_{\text{BACKGROUND}}$ S.T.

$E_8 \to SU(5) \times \underline{SU(5)_\perp}$ — $SU(5)_\perp$ COMMUTANT OF SU(5) IN E_8

$E_8 \to SO(10) \times \underline{SU(4)_\perp}$

TO GET FROM THE GUT GROUP TO (M)SM GROUP:

$SU(5) \to \underbrace{SU(3) \times SU(2) \times U(1)_Y}_{= G_{SM}}$: $\pi_1(M) = \mathbb{Z}_k$ TORSION:

$\pi_1(M) = \mathbb{Z}_k \subset U(1)_Y \subseteq SU(5)$

TURN ON A DISCRETE WILSON LINE $\langle W \rangle = \left(\exp i \oint_C A \right)$

$\Rightarrow$ SU(5) BREAKS TO THE COMMUTANT WHICH IS G_{SM}

TO GET A CY3 W/ $\pi_1(M) \neq 0$: TAKE A QUOTIENT BY $\mathbb{Z}_k$: $\pi_1(M) = 0 \longrightarrow \pi_1(M/\mathbb{Z}_k) = \mathbb{Z}_k$.

<u>CHALLENGES:</u> CONSTRUCT THE SPECTRUM W/O ANY EXOTIC MATTER.

HOW TO DERIVE THE TABLE W/ SPECTRUM OF VM, CM, IN 4D? IN THE NEXT & FINAL LECTURE WE WILL DISCUSS THUS IN DETAIL IN THE CONTEXT OF THE TYPE II STRING.