STRING THEORY II

LECTURE XVI

OXFORD UNIVERSITY
MMATH PHYS TT 2020

② TYPE IIA & IIB 10d STRINGS ON CALABI–YAU THREEFOLDS. M_6

IN 10d: ϵ_1, ϵ_2 ($N=2$) SUSY $\xrightarrow[CY3]{M_6}$ 4d $N=2$

"RECAP": 4d $N=2$ MULTIPLETS.

4d $N=2$ VECTOR MULTIPLET (VM)

$$\begin{array}{ccc} \lambda_\alpha & \longleftrightarrow & A_\mu \\ \Phi & \longleftrightarrow & \tilde{\lambda}_\alpha \end{array}$$

4d $N=1$ DECOMPOSITION

1 $N=1$ VM

$\oplus$ 1 $N=1$ CM (CHIRAL MULTIPLET)

$N=2$ SUSY $\left\{ \begin{array}{l} \text{"} \longleftrightarrow \text{"} \text{ ONE } N=1 \text{ SUSY} \\ \text{ONE } N=1 \text{ SUSY} \end{array} \right.$

4d $N=2$ HYPER MULTIPLET (HM)

$$\begin{array}{ccc} \varphi & \longleftrightarrow & \psi \\ \tilde{\psi}^\dagger & \longleftrightarrow & \tilde{\varphi}^\dagger \end{array}$$

1 $N=1$ CM $\oplus$ 1 $N=1$ CM.

4d $N=2$ GRAVITY MULTIPLET:

$$\underset{\text{spin } 2}{g_{mn}}, \quad \overbrace{\underset{\text{spin } 3/2}{\psi_m, \tilde{\psi}_m}}^{\text{GRAVITINI}}, \quad \overset{\text{VECTOR}}{\underset{\text{spin } 1}{V_m}}$$

THE SPECTRUM OF TYPE II / CY3 TO FALL INTO 4d $N=2$ MULTIPLETS.

KK (KALUZA-KLEIN) REDUCTION ON A CY3: $\mathbb{R}^{1,3} \times M_6$.

GOAL: FIND THE MASSLESS SPECTRUM ALONG $\mathbb{R}^{1,3}$.

10d: $\underset{\mathbb{R}^{1,3}\quad M_6}{\Phi_{\text{Bosonic}}(x, y)}$: $\Delta^{10d}\Phi = 0$

$$\Leftrightarrow \left(\Delta^{\mathbb{R}^{1,3}} + \Delta^{M_6} \right) \Phi = 0 .$$

EIGENVALUES OF Δ^{M_6} ~~SET~~ ACT AS MASSES FOR THE 4d MODES OF Φ.

MASSLESS 4d SPECTRUM IS DETERMINED BY THE ZERO-MODES OF Δ^{M_6}: $\Delta^{M_6}\Phi = 0$

$\Rightarrow$ HARMONIC FORMS ON M_6: $h^{1,1}, h^{1,2}$.

$(h^{p,q})$.

$h^{p,q}$ COUNT THE ZERO MODES IN 4d.

E.G. IN IIA C_3 RR-FORM:

$$C_3 = \sum_{\substack{\text{1-FORMS} \\ \omega^{(1)} \\ \text{ON } M_6}} C_{3\,i}^{(1)} \wedge \omega_i^{(1)} + \sum_{\substack{\text{2-FORM} \\ \omega^{(2)} \\ \text{ON } M_6}} C_{3\,\alpha}^{(2)} \wedge \omega_\alpha^{(2)}$$

$$+ \sum_{\substack{\text{3-FORM} \\ \omega^{(3)} \text{ ON} \\ M_6}} C_{3\,I}^{(3)} \wedge \omega_I^{(3)} .$$

EXPAND INTO P-FORMS ALONG $\mathbb{R}^{1,3}$ & HARMONIC (3-P) FORMS ALONG M_6. [Künneth decomp].

HODGE DIAMOND CY3

$$\begin{array}{ccccccc} h^{p,q} & & & & & & \\ & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & h^{1,1} & & 0 & \\ 1 & & h^{1,2} & & h^{1,2} & & 1 \\ & 0 & & h^{1,1} & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array}$$

$\Rightarrow$ $\omega_i^{(1)}$: ON CY3 ($\neq T^6$ OR $T^2 \times K3$) $\nexists$ HARMONIC 1-FORMS.

$\omega_\alpha^{(2)}$: $h^{1,1}$ 2-FORMS $\Rightarrow$ $C_{3\,\alpha}^{(2)} \hat{=}$ SPACETIME VECTOR FIELD

$\Rightarrow$ $h^{1,1}$ SPACETIME VM.

$$\omega_I^{(3)} : \left. \begin{array}{l} \nearrow h^{1,2} + h^{1,2} = 2h^{1,2} \\ \searrow h^{3,0} + h^{0,3} = 2 \end{array} \right\} \; C_{3I}^{(3)} \text{ 0-FORMS.}$$

$\Rightarrow$ SCALARS IN $\mathbb{R}^{1,3}$.

REPEAT FOR ALL FORM FIELDS C_p & EXPAND

$$C_p = \sum_i \underbrace{C_{p\,i}^{(a)}}_{(p-a)\text{-FORM ALONG } \mathbb{R}^{1,3}} \wedge \omega_i^{(a)}$$

$\omega_i^{(a)}$ = HARMONIC a-FORM.

HARMONIC a-FORMS $\omega_i^{(a)}$ ARE ACCOUNTED FOR BY THE HODGE # : $h^{p,q}$ $\quad p+q=a$.

II A: C_1, C_3, B_2

II B: C_0, C_2, C_4^+, B_2.

METRIC: 10d g_{MN} : $\mathbb{R}^{1,3}$ $g_{\mu\nu}$

M_6: $g_{i\bar{j}} \hat{=} (1,1)$ FORMS.

$h^{(1,1)}$ SCALARS.

$g_{ij}, g_{\bar{i}\bar{j}} \hat{=}$ COMPLEX STRUCTURE

DEFORMATIONS OF THE METRIC: EXPAND $g_{ij}\, g^{j\bar{k}} \underbrace{\Omega_{\bar{k}\bar{\ell}\bar{m}}}$ IN $H^{1,2}$

SIMILARLY FOR $g_{\bar{i}\bar{j}}$ IN $H^{2,1}$.

FROM THESE WE GET $2 \times h^{1,2}$ METRIC DEFORMATIONS

$\Rightarrow$ 4d : SCALARS

Φ : DILATON : 1 SCALAR.

BOSONIC DECOMPOSITIONS ARE SUPPLEMENTED W/ FERMIONS.

$\Rightarrow$ SCALARS COMBINE INTO HM OR VM.

VECTORS HM OR GM.

10 D IIA BOSONIC

$\mu = 0 \ldots 3 \quad \mathbb{R}^{1,3}$

$i, \bar{j} \hat{=} M_6$.

MULTIPLICITY OF ZERO MODES — FORMS ON M_6.

		Multiplicity of zero modes	Forms on M_6
g_{MN}	$\to g_{\mu\nu}$	1 (2 FORM)	$(0,0)$
	$\to g_{i\bar{j}}$	$h^{1,1}$ (SCALARS)	$(1,1)$
	$\to g_{ij}, g_{\bar{i}\bar{j}}$ $\xrightarrow[\Omega^{(0,3)} \text{ CONTRACTION}]{\Omega^{(3,0)}}$	$2h^{1,2}$ (SCALARS)	$(1,2)\oplus(2,1)$
B_{MN}	$\to B_{\mu\nu}$	1 ($B_{\mu\nu}$ ~ SCALAR IN 4d)	$(0,0)$
	$\to B_{i\bar{j}}$	$h^{1,1}$ (SCALARS)	$(1,1)$
Φ		1 (SCALAR)	$(0,0)$
C_1	NO 1FORMS	1 (VECTOR)	$(0,0)$
C_3		$h^{1,1}$ (VECTORS)	$(1,1)$
		$2h^{1,2}$ (SCALARS)	$(1,2)\oplus(2,1)$
		2 (SCALARS)	$(3,0)\oplus(0,3)$.

1 x 4d N=2 GRAVITY MULTIPLET

$h^{1,2}$ HM.

$h^{1,1}$ VM.

1 HM.

TO SEE THE MULTIPLET STRCT: SUSY DECOMPOSITION

$\boxed{\begin{array}{l} \text{IIA ON CY3} \\ (h^{1,1}, h^{1,2}) \\ 1 \text{ GM} \\ h^{1,2}+1 \quad \text{HM} \\ h^{1,1} \quad \text{VM} \end{array}}$

SIMILARLY FOR IIB:

- G, B, Φ ARE THE SAME.
- C_0, C_2, C_4^+ DECOMPOSITION AS ON SLIDE 4.

IIB ON CY3 $(h^{1,1}, h^{1,2})$

1 GRAVITY MULTIPLET

$h^{1,1}+1$ HM

$h^{1,2}$ VM

IIA ON CY3 $(h^{1,1}, h^{1,2})$

1 GRAVITY MULTIPLET.

$h^{1,2}+1$ HM

$h^{1,1}$ VM.

CANDELAS, DE LA OSSA:

CY3 M_6: $h^{1,1}(M_6)$

$h^{1,2}(M_6)$

CY3 W_6: $h^{1,1}(W_6)$

$h^{1,2}(W_6)$

IF: $h^{1,1}(W_6) = h^{1,2}(M_6)$

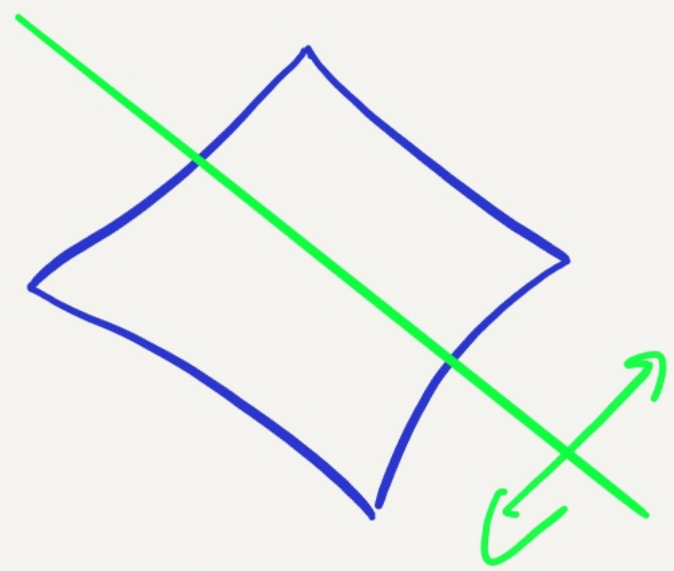
$h^{1,2}(W_6) = h^{1,1}(M_6)$.

$\Rightarrow$ $\circledast$ IIB ON W_6 $\cong$ IIA ON M_6.

SAME SPECTRUM

$$\begin{matrix} VM & & HM \\ HM & & VM \end{matrix}$$

$h^{1,1} \leftrightarrow h^{1,2}$



MIRROR SYMMETRY.

M_6 & W_6 SATISFYING $h^{11}(M_6) = h^{12}(W_6)$ $h^{1,2}(M_6) = h^{1,1}(W_6)$
ARE CALLED MIRROR CALABI-YAU MANIFOLDS.
THERE IS LOTS EVIDENCE THAT THE STRING THEORIES IN $\circledast$ ARE EQUIVALENT!

- SUGRA
- WORLSHEET ARGUMENT.

WORLDSHEET POINT OF VIEW:

1) MIRROR SYMMETRY IS A GENERALISATION OF T-DUALITY, WHICH IS AN EXACT SYMMETRY OF THE 2d CFT.

E.G. SOME CY HAVE A DESCRIPTION AS A QUOTIENT OF T^6: $\quad M_6 = T^6/\Gamma$ "ORBIFOLD".

Γ DISCRETE GROUP, E.G. $\mathbb{Z}_3$

$\Rightarrow$ "ORBIFOLD" IS AN EXACT CFT OPERATION: SPECTRUM, INVARIANT & "COVARIANT" (TWISTED SECTOR) STATES & INTERACTIONS.

$\Rightarrow$ T-DUALITY OF T^6 INDUCES MIRROR SYMMETRY OF M_6.

2) GEPNER-MODELS: CFT CONSTRUCTIONS OF CY-COMPACTIFICATIONS IN THE QUANTUM REGIME $\Rightarrow$ MIRROR SYMMETRY IS A SYMMETRY OR GEPNER MODELS.

ANOTHER WAY TO PHRASE THE MIRROR SYMMETRY: OPEN STRINGS, I.E. DBRANES. [MIRROR SYMMETRY: AMS].

$\rightsquigarrow$ D-BRANES IN CY-COMPACTIFICATIONS ARE OBJECTS IN CATEGORIES. EQUIVALENCE OF THESE D-BRANE CATEGORIES IS A HOT TOPIC OF MATHEMATICAL RESEARCH.

MORE BROADLY: MIRROR SYMMETRY STARTED AS CURIOUS OBSERVATION IN STRING TH, EVOLVED INTO ITS OWN PURE MATH SUBJECT ∇: EXAMPLE OF HOW STRINGS CAN PROVIDE CONJECTURES & SURPRISING CONNECTIONS BETWEEN SUBJECTS.

MANY TOPICS WE DID NOT DISCUSS:

- HOLOGRAPHY: WE TOUCHED UPON AdS - SUGRA SOLUTIONS.

$$AdS_d \times M_{10-d} \longleftrightarrow \text{CONFORMAL FIELD THEORY IN } d-1 \text{ DIM } (\partial AdS_d.)$$

$AdS_5 \times S^5 \longleftrightarrow$ 4d MAX SYM $U(N) = G$

HOLOGRAPHY GIVES ONE WAY ACCESS YM THEORIES AT STRONG COUPLING (BEYOND WHAT WE LEARN FROM QFT BOOKS).

$\rightsquigarrow$ STRONG COUPLING IS DESCRIBED BY STRING TH IN A SPACETIME W/ AN AdS- FACTOR.

$\rightsquigarrow$ SPECTRUM OF 4d $N=4$ (MAX) SYM & $AdS_5 \times S^5$ IIB STRINGS IS KNOWN TO AGREE TO A VERY HIGH ACCURACY.

CONSTRUCTIONS OF SUPER CONFORMAL FIELD TH. (SCFT)

WE STUDIED 2d SCFTs (WS OF IIB, A STRINGS).

IN HIGHER DIM: SCFTs EXIT UP TO & INCL. DIM 6. (NAHM).

CONSTRUCT HIGHER DIM SCFTs & IN DIM 5 & 6 THESE STRONGLY COUPLED THEORIES (SCFT "UV") $g \to \infty$.

$\Rightarrow$ NO PERTURBATIVE METHODS.

(IN 4d: COMPUTE β-FUNCTION: $\beta = 0$ CFT).

CONSTRUCT THESE FROM COMPACTIFICATIONS OF
IIB + τ (AXIO-DILATON) $\equiv$ F-THEORY $\rightsquigarrow$ 6d SCFTs
M-THEORY ON CY-3-FOLDS $\rightsquigarrow$ 5d SCFTs.
SCALE INVARIANCE OF THE SCFT $\Rightarrow$ GEOMETRY
NEEDS TO BE SINGULAR.

$\Rightarrow$ CY 3-FOLD SINGULARITIES $\longleftrightarrow$ 6d & 5d SCFTS.

EXPLORATION OF CY 3-FOLDS W/ HELP OF SCFTs
& VICE VERSA.