String Theory II: Assignment 3

(1) Super-Yang Mills in various dimensions

Let A_μ be a 10d $U(N)$ gauge field and λ a 16 component Majorana-Weyl spinor (gaugino) in 10d [recall: spinor costruction in the lectures 3/4. Majorana means we impose a reality condition $\lambda^* = B\lambda$.]

1. Show that
$$\mathcal{L}_{10dSYM} = -\frac{1}{4g_{YM}^2}\,\mathrm{Tr}\,F_{\mu\nu}F^{\mu\nu} - \frac{i}{2g_{YM}^2}\,\mathrm{Tr}(\bar{\lambda}\Gamma^\mu D_\mu\lambda) \tag{1}$$
is invariant under the supersymmetry transformations with the supersymmetry parameter ϵ
$$\begin{aligned}\delta A_\mu &= -i\bar{\epsilon}\Gamma_\mu\lambda \\ \delta\lambda &= \frac{1}{2}F_{\mu\nu}\Gamma^{\mu\nu}\epsilon\end{aligned} \tag{2}$$

2. By Kaluza-Klein reducing along T^d, and thererby decomposing
$$SO(1,9) \to SO(d) \times SO(1,9-d) \tag{3}$$
(i.e. splitting the indices μ into sets along d compact dimensions and $10-d$ non-compact dimensions) determine the Lagrangian in $10-d$ dimensions and the supersymmetry transformations. [Hint: you will have to decompose the spinors according to (3).]

(2) Free Fermion Description of the Heterotic String

Consider the Vertex operator algebra generated for $\mu = 0, \cdots, 9$ by

- $X^\mu(z, \bar{z})$ be free bosonic fields (non-chiral)
- $\lambda^A(z)$, $A = 1, \cdots, 32$ be holomorphic world-sheet fermion fields with periodic boundary conditions.
- $\tilde{\psi}^\mu(\bar{z})$ be anti-holomorphic world-sheet fermion fields

The OPE algebra is thus

$$\begin{aligned} X^\mu(z, \bar{z}) X^\nu(0, 0) &\sim -\eta^{\mu\nu} \frac{\alpha'}{2} \ln |z|^2 \\ \lambda^A(z) \lambda^B(0) &\sim \delta^{AB} \frac{1}{z} \\ \tilde{\psi}^\mu(\bar{z}) \tilde{\psi}^\nu(0) &\sim \eta^{\mu\nu} \frac{1}{\bar{z}} \,. \end{aligned} \tag{4}$$

1. Compute the OPEs TT and $\bar{T}\bar{T}$, where

$$\begin{aligned} T(z) &= -\frac{1}{\alpha'} \partial X^\mu \partial X_\mu - \frac{1}{2} \lambda^A \partial \lambda^A \\ \bar{T}(\bar{z}) &= -\frac{1}{\alpha'} \bar{\partial} X^\mu \bar{\partial} X_\mu - \frac{1}{2} \tilde{\psi}^\mu \bar{\partial} \tilde{\psi}_\mu \end{aligned} \tag{5}$$

2. Using bosonization of the left moving fermions λ^A show that this CFT is equivalent to the 10d $SO(32)$ heterotic string.
3. Construct in the fermionic description (i.e. using λ^A) the mass-less states, taking care of imposing a GSO projection on both left and right-moving sectors.
4. Choose now boundary conditions for 16 of the λ^A, which are anti-periodic. Show that these fields realize a level 1 $SO(16) \times SO(16)$ current algebra. Using bosonization, construct in terms of λ^A the current algebra that realizes the 10d $E_8 \times E_8$ heterotic string. [Hint: you will need to construct momentum VO that extend the manifest symmetry $SO(16) \times SO(16)$ to $E_8 \times E_8$.]