M Sc in Mathematical and Computational Finance

Quantitative Risk Management: Problem sheet 1

1. Read the chapter from Jarrow's text 'The Economic Foundations of Risk Management' on the 1993 failure of Metallgesellschaft (MG).

   - Summarize the key risk factors in the MG strategy.
   - Which of these risk factors could be addressed using historical models?
   - Summarize how MG thought it was managing risk, and how their actions did and did not work.

2. Consider a portfolio made up of one unit of a stock S, and some number A of put options with strike K and expiry T. In what follows this portfolio is static (i.e. it is entered into at time zero, and no modification of the position is possible afterwards). The initial prices of stock and options are S_0, P_0 respectively.

   i. Compute the smallest number of options A which minimizes the worst-case risk of the losses on this portfolio at time T.

   ii. For a given K, compute the number of options A which makes the portfolio Δ-neutral at time 0, assuming the Black–Scholes pricing model.

   iii. For a fixed $A > 1$, write down an formula which determines the value-at-risk at level 0.95 over horizon T in terms of the standard normal CDF, assuming the Black–Scholes model holds with $\mathbb{P}$-drift μ constant. (Hint: First find the set of prices of the stock for which the portfolio has value $\leq B$ for each B.)

   iv. Using implicit differentiation or otherwise, find the sensitivity of this value at risk to the underlying volatility σ. (Assuming the price of the option remains fixed)

3. i. Write a python script which will take:

   - A list of observed values
   - A high 'threshold' value u

   And returns the maximum likelihood estimates of β, ξ in a Generalized Pareto model using a peaks-over-threshold approach. See Slide 109 for the log-likelihood function. If using `scipy.optimize.minimize` for numerical optimization, you may also wish to consider the inverse hessian of the negative log-likelihood (as this is a crude estimate of the standard error covariance matrix of the estimated parameters).

ii. Simulate 100 observations from a standard t-distribution with 4 degrees of freedom, and use maximum likelihood to estimate the parameters (β, ξ) using observations above $u = 1.5$.

iii. Apply Smith's tail estimator (slide 116) to compute the Value at Risk at levels $\alpha = 0.95, 0.99, 0.999$, and compare this with the true value.

iv. Repeat the simulation/estimation steps above to generate a histogram of the distribution of Value at Risk estimates at level 0.99.