M Sc in Mathematical and Computational Finance

Quantitative Risk Management: Problem sheet 1

1. Read the chapter from Jarrow's text 'The Economic Foundations of Risk Management' on the 1998 failure of Long Term Capital Management (LTCM).

   - Summarize the key risk factors in the LTCM strategy.
   - LTCM's strategy depended on diversification of risks. Explain how the diversification rule described by Jarrow, which focuses on variance calculations, can be misleading in extreme situations.
   - Describe the relationship between the market risk and funding risks faced by LTCM.

2. Suppose we have two assets (X, Y) with returns described by marginal cdfs F_X, F_Y and a copula C.

   (i) Show that the probability that the sum of X and Y is less than or equal to k is given by

   $$\int_{-\infty}^{k} \frac{C(F_X(x), F_Y(k-y))}{C(F_X(x), 1)} F_X(dx)$$

   (ii) Suppose X and Y have negative tails with generalized pareto distributions, that is,

   $$F_X(z) = (1 + \xi z)^{1/\xi} \qquad \text{for all } z \text{ sufficiently small}$$

   for some $\xi \neq 0$, and similarly for Y (with the same value of ξ). Give a formula (implicitly defined in terms of an integral) to compute the value at risk of $X + Y$ at level α, in terms of ξ and θ.

   (iii) Write down the density of the Clayton copula in this setting.

3. Consider observations of asset returns $\mathbf{X} = (X_1, ..., X_d)$ from an elliptical distribution $M(\mu, \Sigma, \psi)$.

   (i) For a fully invested portfolio $\pi \in \mathbb{R}^d$ with $\sum \pi_i = 1$, state the distribution of the portfolio return $\pi^\top \mathbf{X}$.

   (ii) Give a closed-form expression for the value at risk, if ψ is from a standard Laplace distribution.

   (iii) How would changing the value of ψ alter the choice of minimum value-at-risk portfolio?

4. Implement a python script which:

   (i) Downloads historical prices 2015-2019 of Facebook, Amazon, Google

   (ii) Fits a t-distribution to the log-returns of each stock (using the standard t distribution fit method)

   (iii) Fits a Clayton copula to the joint returns (transformed using the t-distribution fitted in (ii)) (you may need to hand-code this)

   (iv) Estimates the probability that all three stocks have returns below their worst 1% quantile on any given day, using this copula.

   (v) Compare this with what one might expect, from observing the Pearson correlation of the returns series.