Example of a LaTeX Document

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1 Introduction

Mathematical models based on partial differential equations are being used to an increasing extent to model the world about us. They can be exploited to describe many different phenomena in areas ranging from fluid mechanics to finance, from tumour growth to traffic flow, and from economics to our own special interest, electrochemistry. Of course a variety of analytical techniques exist for solving partial differential equations exactly, such as Laplace or Fourier transform methods, similarity solution methods, series solutions via separation of variables and Green's function techniques. However, except in the most simple cases, these analytical techniques are either inapplicable or impractical; indeed for two-dimensional problems in electrochemistry the only problem to which there is a known closed form solution is the simple case of an E reaction mechanism at a microdisc electrode. Of course the more interesting situations are modelled more realistically by complicated equations so we need to resort to numerical methods to solve such problems. Two obvious candidates for suitable methods are the finite difference method (FDM) and the finite element method (FEM).

1.1 Finite Difference Methods

Finite difference methods date back to the 1920's and are based on two ideas. First, one approximates the computational domain by a finite set of points (known as mesh points, grid points or nodes), then one approximates the differential equation using difference equations. Clearly this leads to a linear or nonlinear system of algebraic equations to find the approximate solution at the mesh points. The hope is that as the computational mesh is refined, the approximate solution will converge to the exact solution; indeed, provided the solution is sufficiently smooth, error estimates can be derived to show that this is the case and they indicate the expected rate of convergence. Although the basic ideas of finite difference methods are relatively straightforward, there are drawbacks. The first is that the method does not easily handle domains with curved or irregular boundaries, another is that there are various ways to handle boundary conditions and it is not obvious, in advance, which is the best. This means that it is difficult to write a generally applicable computer code based on the finite difference method.

1.1.1 Finite Difference Methods for the Heat Equation

Now we consider various methods for solving the heat equation in one space dimension. We will consider the explicit Euler scheme and the implicit Euler scheme. We will then take the average of these two methods to produce the Crank-Nicolson scheme. In fact we will see that all of these schemes are special cases of the so-called "theta-method".

2 Random Maths

( ) : [ ] . !

$ & % # _ { }

The equation $\alpha_1 = \beta^2$ tells us

The equation $\alpha_1 = \beta^2$ tells us

The equation $\alpha_1 2 = \beta^2$ tells us

$$\alpha_1 = \beta^2$$

$$\alpha_1 = \beta^2$$

$$\alpha_1 = \beta^2 \tag{1}$$

$$\alpha_1 = \beta^2 \tag{2}$$

Equation (2) explains the relationship between α_1 and β.

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$$\begin{aligned} \beta &= 1 && (3) \\ \alpha_1 &= \beta^2 && (4) \\ \alpha_2 &= \beta^3 && (5) \end{aligned}$$

$$\begin{aligned} \beta &= 1 && (6) \\ \alpha_1 &= \beta^2 \\ \alpha_2 &= \beta^3 && (7) \end{aligned}$$

$$\begin{aligned} \beta &= 1 \\ \alpha_1 &= \beta^2 \\ \alpha_2 &= \beta^3 \end{aligned}$$

$$
\begin{aligned}
I &= \int_1^\infty x^{-1}\mathrm{d}x \\
S &= \sum_{n=0}^{s} n \\
f &= \frac{3}{2x} \\
u_x &= \frac{\partial u}{\partial x} \\
&\sim \cos \tan \log \exp \max min
\end{aligned}
$$

Remember not to just use min in mathmode — this is formally the product of m, i and n.

$$
A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \tag{8}
$$

$$
|x| = \begin{cases} -x & x < 0 \\ x & x \geq 0 \end{cases}
$$

$$
\begin{aligned}
\frac{\partial u}{\partial x} &= x^2 + \sin x \\
&\quad + \cos x \qquad (9)
\end{aligned}
$$

$$
\frac{\partial u}{\partial x} = x^2 + \sin x + \cos x = 4 \tag{10}
$$

$$
\begin{aligned}
\frac{\partial u}{\partial x} &= x^2 + \sin x + \cos x & (11) \\
&= 4 & (12)
\end{aligned}
$$

3 Tables

In Table 1 we see some numbers.

mesh	triangles	nodes	current
1	32	25	1.270
2	94	59	1.131
3	201	116	1.066
4	372	208	1.034
5	527	288	1.019

Table 1: Triangulations produced by FEM.

mesh	triangles	nodes	current
1	32	25	1.270
2	94	59	1.131
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Table 2: Triangulations produced by FEM.

4 Figures

In Figure 1 we see a sphere.

In Figure 2 we see

5 Theorems

Theorem: Picard. *Suppose that $f(t,u)$ is a continuous function of t and u in a region $\Omega = [0,T) \times [u_0 - \alpha, u_0 + \alpha]$ of the (t,u) plane and that there exists $L > 0$ such that*

$$|f(t,u) - f(t,v)| \quad \leq \quad L|u - v| \quad \forall t, u, v \in \Omega.$$

L is called a Lipschitz constant and this a Lipschitz condition. Suppose also that

$$MT \quad \leq \alpha,$$

where $M = \max_\Omega |f|$. Then there exists a unique continuously differentiable function $u(t)$ defined on $[0,T)$ satisfying

$$\begin{aligned} \frac{\mathrm{d}u}{\mathrm{d}t} &= f(t,u), \quad 0 < t < T \\ u(0) &= u_0. \end{aligned}$$

Proof. prove the theorem here □

Proof. prove the theorem another way here □

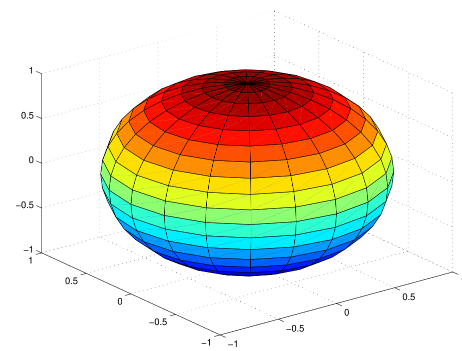

Figure 1: A sphere.

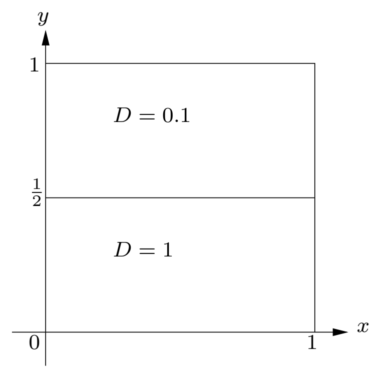

Figure 2: Variation of diffusion coefficients.

6 Code

```
% file mynewt.m
% this function finds a root of sin(x)-cos(x)+exp(-x)
% using Newton's method
function x=mynewt(xguess)

f=@(x) sin(x)-cos(x)+exp(-x);
fprime=@(x) cos(x)+sin(x)-exp(-x);

x=xguess;
tol=1e-10;

while abs(f(x)) > tol
    x=x-f(x)/fprime(x)
end
end
```

7 Referencing

For an introduction to finite difference methods see [1].

References

[1] K.W. Morton and D.F. Mayers. Numerical Solution of Partial Differential Equations. Cambridge University Press, 1994.

[2] P. Houston, J. Mackenzie, E. Süli and G. Warnecke. A posteriori error analysis for numerical approximations of Friedrichs systems. *Numerische Mathematik*, **82**:409–432, 1999.