Practical Numerical Analysis: Sheet 2

1. In this problem we will approximate the values of four integrals

$$\text{(a)} \quad \int_0^1 4\pi x \sin(20\pi x)\cos(2\pi x)\mathrm{d}x \;=\; -20/99\;,$$

$$\text{(b)} \quad \int_0^1 \sin(2\pi x)\cos(4\pi x)\mathrm{d}x \;=\; 0\;,$$

$$\text{(c)} \quad \int_0^5 G(x)\mathrm{d}x \;=\; 7.5\;, \qquad \text{where } G(x) \;=\; \begin{cases} x+1 & x<1, \\ 3-x & 1\le x\le 3, \\ 2 & x>3. \end{cases}$$

$$\text{(d)} \quad \int_0^1 x^{3/2}\mathrm{d}x \;=\; 0.4\;.$$

Note that in Matlab you can implement $G(x)$ as:

```
G=@(x) (x+1).*(x<1)+(3-x).*(1<=x).*(x<=3)+2*(x>3);
```

Compute each integral using the composite trapezium rule, the Clenshaw-Curtis rule and a Gauss-Legendre rule with `n=10:10:100`.

Note that the Legendre polynomials are the orthogonal polynomials on $[-1,1]$ with the unit weight function. The orthonormal Legendre polynomials are defined by $P_0(x) = 1/\sqrt{2}$, $P_1(x) = \sqrt{3/2}x$ and

$$xP_n(x) \;=\; \frac{1}{2}\frac{1}{\sqrt{1-1/(2(n+1))^2}}P_{n+1}(x) + \frac{1}{2}\frac{1}{\sqrt{1-1/(2n)^2}}P_{n-1}(x)\;,$$

for $n = 1, 2, \ldots$.

For each integral produce a plot showing the convergence of the error with n for each of the three different methods.

2. Use Romberg integration to compute the integral in 1(a) accurately.

Further Reading

1. E. Süli and D. Mayers, *An Introduction to Numerical Analysis*, CUP, 2003.
2. L.N. Trefethen, *Is Gauss Quadrature Better than Clenshaw-Curtis?*, SIAM Review, vol 50, pages 67–87, 2008.
3. N. Hale and A. Townsend, *Fast and Accurate Computation of Gauss-Legendre and Gauss-Jacobi Quadrature Nodes and Weights*, SISC, vol 35, pages A652–A674, 2013.