Practical Numerical Analysis: Sheet 5

1. Solve the boundary value problem

$$u'' = -4\pi^2 u$$

on the interval $[-1, 1]$ with boundary conditions $u(-1) = 0$ and $u'(1) = 2\pi$.

Use both a spectral collocation method with Chebyshev nodes and a standard three point finite difference scheme (on a uniform mesh) on grids with 4, 8, 16, 24, 32 and 48 mesh spacings. Plot (using `semilogy`) the maximum absolute difference between the computed values of $u(x)$ at the nodes and the exact solution $\sin(2\pi x)$ for both methods as a function of the number of nodes.

Further Reading

1. Lloyd N. Trefethen, *Spectral Methods in Matlab*, SIAM, 2000.
   (A compact introduction to Spectral Methods with many examples in Matlab. Differentiation Matrices are described in Chapter 6, boundary value problems in Chapter 7.)

2. Bengt Fornberg, *A Practical Guide to Pseudospectral Methods*, CUP, 1998.
   (Another excellent introduction to spectral methods. Finite difference stencils are discussed in Chapter 3, and differentiation matrices in Chapter 4.)