### **Message Authentication Code**



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### Outline



### 2 Message Authentication Code (MAC)

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- We want parties to *securely* communicate over insecure channels.
- Encrypting messages is only one part of security.
- What if the messages were modified in transit?
- What about authenticity?

# Encryption does not guarantee integrity

- Consider the OTP scheme, which is a perfectly secret encryption scheme.
- From a given ciphertext, you can produce a new *valid* ciphertext, by just flipping a single bit!
- Perfect secrecy is not violated here.
- But, perfect secrecy *simply* doesn't imply message integrity.
- Different cryptographic tools should be used to achieve secrecy and integrity.

## Outline



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## Message Authentication Code (MAC)

- Message authentication code is the cryptographic tool to be used to ensure message integrity.
- Informally speaking, the MAC's goal is to prevent an adversary from tampering with the messages.
- Parties need to share a secret key as in the encryption!

#### Definition

A MAC consists of the following three probabilistic polynomial-time algorithms (KeyGen, Mac, Verify):

- KeyGen(1<sup>n</sup>): takes the security parameter n and outputs a key k
   s.t. |k| ≥ n
- Mac<sub>k</sub>(m ∈ {0,1}\*): is a tagging algorithm, takes a key k and a message m and outputs a tag t.
- Verify<sub>k</sub>(m, t): a deterministic algorithm that outputs a bit b, 0 for invalid and 1 for valid.

 $Mac_k(\cdot)$  may be randomised or deterministic.

- Correctness of MAC:  $\forall n, \forall k \leftarrow \text{KeyGen}(1^n) \text{ and } \forall m \in \{0, 1\}^*$ , Verify<sub>k</sub>(m, Mac<sub>k</sub>(m)) = 1 holds.
- Fixed-length MAC: if it is just defined for messages m ∈ {0,1}<sup>ℓ(n)</sup>, we call the scheme a *fixed-length MAC* for messages of length ℓ(n).
- Canonical Verification: when Mac is deterministic, recomputes the tag and checks for equality.

- An adversary should not be able to efficiently produce a valid tag on a new message that was not authenticated before.
- Taking as realistic a scenario where the adversary can see message/tag pairs, in the security definition the adversary is given access to a tagging oracle.

# **Security of MAC - Formal Definition**

Given S = (KeyGen, Mac, Verify), an adversary A, and a security parameter n, we define the following experiment:

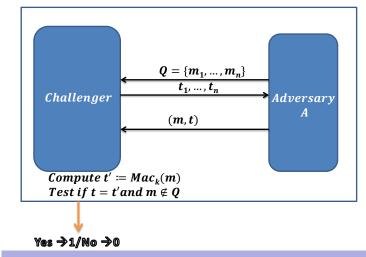
### **Experiment** (Mac $_{A,S}^{Unforg}$ )

- Key generation:  $k \leftarrow \text{KeyGen}(1^n)$ .
- Tag queries: the adversary A is given oracle access to Mac<sub>k</sub>().
   The set of all queried messages is Q.
- Adversary's output: the adversary A eventually outputs (m, t)
- Experiment's output: if

$$\mathsf{Verify}_k(m,t) = 1 \land m \not\in Q$$

outputs 1, otherwise outputs 0.

MAC<sup>unforg</sup> Game



A MAC scheme *S* is said to be *Existentially unforgeable under an adaptive chosen-message attack* if no PPT adversary A can win the previous game with non-negligible probability:

#### Definition

A message authentication code S = (KeyGen, Mac, Verify) is secure if for all probabilistic polynomial-time adversary A, the following holds

$$\Pr[\mathsf{Mac}_{\mathcal{A},S}^{\mathsf{Unforg}}(n) = 1] \le \mathsf{negl}(n)$$
.

- An adversary cannot change the message without being detected by the receiver if it has a valid tag.
- However, the adversary can replay and send the same message again, with the same tag.
- The receiver cannot detect this malicious behaviour.
- Common techniques to prevent replay attacks:
  - Time-stamps: add the current time to the beginning of the message before authenticating it.
  - Counters: users maintain synchronised state.

# Security of MAC - Formal Definition (2)

Given S = (KeyGen, Mac, Verify), an adversary A, and a security parameter n, we define the following experiment:

### **Experiment (Mac**<sup>s-unforg</sup>)</sup>

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   The set of all pairs queried message/tag is Q.
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- Experiment's output: if

$$\mathsf{Verify}_k(m,t) = 1 \land (m,t) \not\in Q$$

outputs 1, otherwise outputs 0.

If a MAC scheme is strongly secure, then adversaries win if they produce tags on any messages (including already authenticated ones!).

#### Definition

A message authentication code S = (KeyGen, Mac, Verify) is strongly secure if for all probabilistic polynomial-time adversary A, the following holds

$$\Pr[\mathsf{Mac}_{\mathcal{A},S}^{\mathsf{s}-\mathsf{unforg}}(n)=1] \le \mathsf{negl}(n) \,.$$

If the Mac algorithm in S is deterministic, and the verification is canonical, then secure MACs are strongly secure as well.

- When giving the adversary access to a MAC oracle, he just learns the output, not the time taken by the Oracle to perform the task.
- This is not what happens in the real systems!
- An adversary may be able to obtain the time necessary to reject a pair message/tag.
- In the case of deterministic MAC, if the MAC verification does not use time-independent string comparison, then the adversary can exploit the time differences to deduce new bytes of the tag!
- This is a realistic attack. Xbox 360 had a difference of 2.2 milliseconds in comparing j or j + 1 bytes.
- Attackers managed to exploit this.
- Conclusion: MAC verification should compare all the bytes.

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#### Definition

Given a length-preserving pseudorandom function F, a fixed-length MAC S for messages of length n consists of the two following algorithms:

- $Mac(k \in \{0,1\}^n, m \in \{0,1\}^n)$ : it outputs the tag  $t \leftarrow F_k(m)$ .
- Verify $(k \in \{0, 1\}^n, m \in \{0, 1\}^n, t \in \{0, 1\}^n)$ : it outputs 1 iff  $F_k(m) = t$

If  $|m| \neq |k|$ , then Mac outputs  $\perp$  and Verify outputs 0.

# A fixed-Length MAC from a PRF

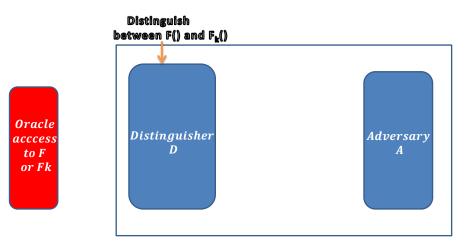
#### Theorem

If *F* is a secure pseudorandom function, then the fixed-length MAC for messages of length n is secure.

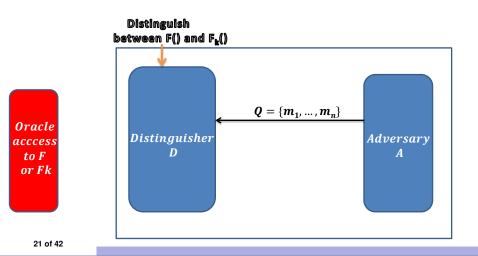
#### Steps of the proof:

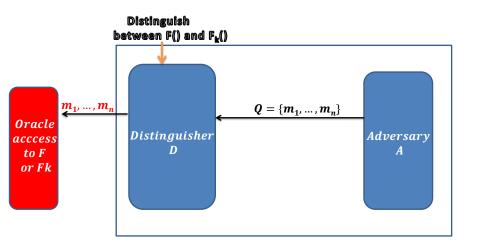
- consider a variation S' of S, where  $F_k$  is replaced by a truly random function  $f : \{0, 1\}^n \to \{0, 1\}^n$ .
- Let A be the adversary trying to attack S.
- Define a distinguisher *D* for *F* (it is given access to some function and needs to tell whether this function is pseudorandom or truly random).
- *D* emulates the MAC experiment for *A* and check if it succeeds in producing a valid tag on a new message *m*.
- if *A* manages to produce a valid tag, *D* will guess that its oracle is "pseudo-random" (1), otherwise it outputs "truly random" (0).

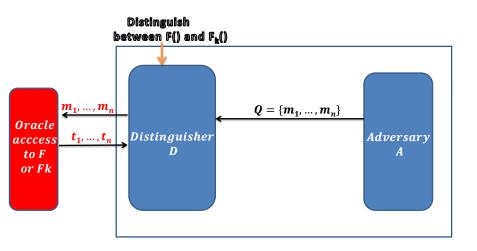
F in the left box is the truly random function  $f : \{0,1\}^n \to \{0,1\}^n$ .

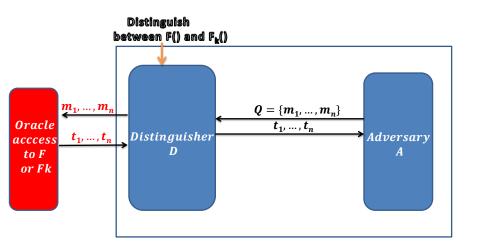


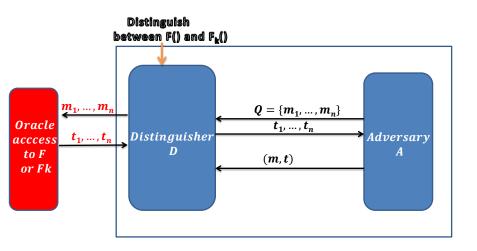
In the "adaptive" setting, the messages  $m_1, \ldots, m_n$  will be sent separately.

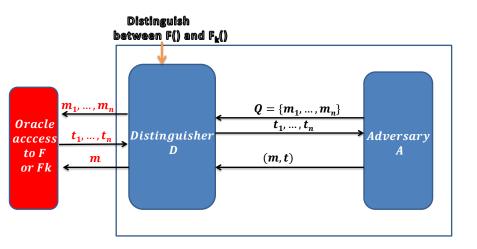


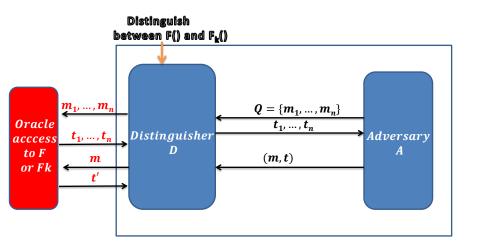


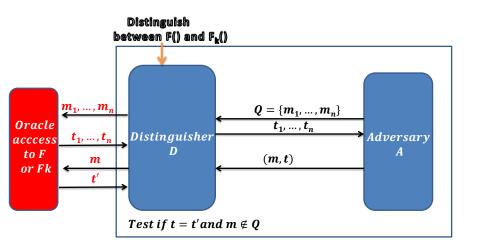


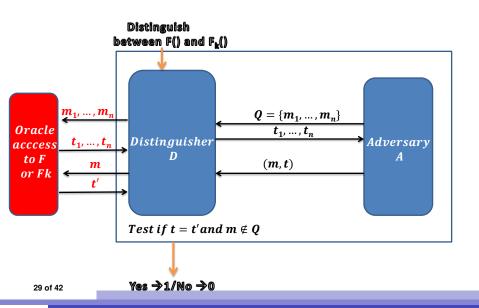










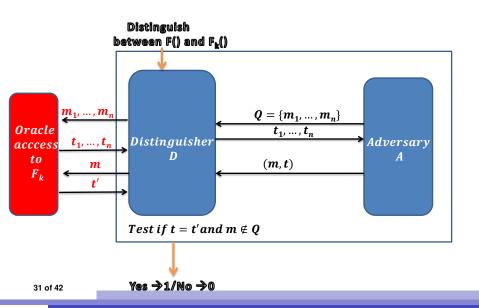


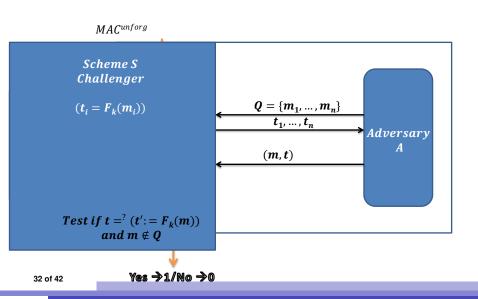
### A fixed-Length MAC from a PRF

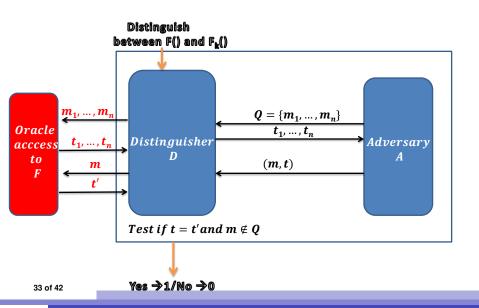
#### **Steps of the Proof**

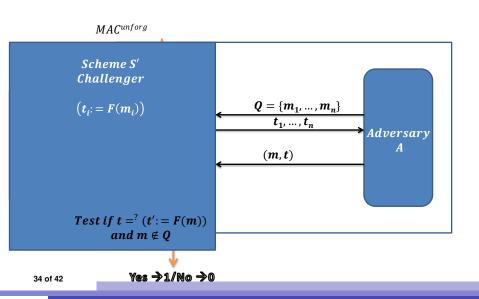
We can distinguish between two cases:

- *D*'s oracle is a pseudo-random function: in this case, the view of A that is run as a subroutine by D and its view in the experiment  $Mac_{A,S}^{Unforg}(n)$  are distributed identically. Moreover, D outputs 1 exactly when  $Mac_{A,S}^{Unforg}(n)$  outputs 1.
- D's oracle is a truly-random function: in this case, the view of A that is run as a subroutine by D and its view in the experiment Mac<sup>Unforg</sup><sub>A,S'</sub>(n) are distributed identically. Moreover, D outputs 1 exactly when Mac<sup>Unforg</sup><sub>A,S'</sub>(n) outputs 1.









#### Sketch Proof.

As a result, we have that

$$\Pr[\mathsf{Mac}_{\mathcal{A},S'}^{\mathsf{Unforg}}(n) = 1] = \Pr[D^{f()}(n) = 1]$$
(1)

and

$$\Pr[\mathsf{Mac}_{\mathcal{A},S}^{\mathsf{Unforg}}(n) = 1] = \Pr[D^{F_k()}(n) = 1]$$
(2)

### A fixed-Length MAC from a PRF

#### Steps of the Proof

Since F is a secure PRF, it holds:

$$|\Pr[D^{f()}(n) = 1] - \Pr[D^{F_k()}(n) = 1]| =$$

 $= |\Pr[\mathsf{Mac}_{\mathcal{A},S'}^{\mathsf{Unforg}}(n) = 1] - \Pr[\mathsf{Mac}_{\mathcal{A},S}^{\mathsf{Unforg}}(n) = 1]| \leq \mathsf{negl}(n).$ 

For any message  $m \notin Q$ , the value t = f(m) is uniformly distributed in  $\{0, 1\}^n$  from the point of view of the adversary A. So:

$$\Pr[\mathsf{Mac}_{\mathcal{A},S'}^{\mathsf{Unforg}}(n) = 1] \le 2^{-n}.$$

The relations above then give:

$$\Pr[\mathsf{Mac}_{\mathcal{A},S}^{\mathsf{Unforg}}(n) = 1] \le 2^{-n} + \mathsf{negl}(n) \,.$$

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# From fixed length MAC to MAC for arbitrary-length messages.

- If the PRF has a bigger bolck length, the MAC is secure for longer messages.
- Problem: existing pseudo-random functions used in practice (block ciphers) can just take short fixed-length inputs!
- Question: How to build a MAC for arbitrary-length messages?

- Block re-ordering attack: change the order of blocks. Namely, if  $(t_1, t_2)$  is a valid tag on  $(m_1, m_2)$  where  $m_1 \neq m_2$ , then  $(t_2, t_1)$  is a valid tag on  $(m_2, m_1)$ , with  $m_1, m_2 \neq m_2, m_1$ . Solution: authenticate a block index with each block.
- Truncation attack: the attacker removes blocks from the end of the message and their corresponding blocks from the tag. Solution: authenticate the message length with each block
- Mix-and-match attack: given the valid tags  $(t_1, t_2, t_3)$  and  $(t'_1, t'_2, t'_3)$  on the messages  $(m_1, m_2, m_3)$  and  $(m'_1, m'_2, m'_3)$ , output  $(t_1, t'_2, t_3)$  on the message  $(m_1, m'_2, m_3)$ . Solution: authenticate a *random message identifier* along with each block.

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#### A MAC from a fixed-length one

#### Definition

Let  $S_1 = (\text{KeyGen}_1, \text{Mac}_1, \text{Verify}_1)$  be a fixed-length MAC for messages of length *n*. We define a MAC *S* for arbitrary-length messages as follows:

• Mac(k ∈ {0,1}<sup>n</sup>, m ∈ {0,1}\*):

- it takes a key k and a messge m, where  $|m| = \ell < 2^{n/4}$ .
- *it then parses* m *into* d *blocks of length* n/4, *i.e.*  $m_1, \dots, m_d$ .
- if the last block is not of size n/4, we pad it with 0s
- *it uniformly chooses*  $r \in \{0, 1\}^{n/4}$
- For  $i = 1, \dots, d$ , compute  $t_i \leftarrow Mac_1(k, r||\ell||i||m_i)$ , where  $i, \ell$  are encoded as strings of length n/4.
- *Output*  $t = (r, t_1, \cdots, t_d)$ .
- Verify $(k, m, (r, t_1, \dots, t_d))$ : parse m into d' blocks, then output 1 iff d' = d AND Verify $_1(k, r||\ell||i||m_i, t_i) = 1$  for  $1 \le i \le d'$ .

#### Theorem

If  $S_1$  is a secure fixed-length MAC for messages of length n, then S as defined above is a secure MAC for arbitrary-length messages.

Another way to build a secure MAC for arbitrary-length messages is to use hash functions, which will be covered soon!

#### Further Reading (1)

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