

Preamble

This sheet is split into two parts. Problems 1-2 may be attempted after the lesson of Tuesday 05/11/2019 and problems 3-6 should be attempted after the lesson of Thursday 07/11/2019.

Questions

Problem 1

Prove that the following modifications of CBC-MAC do not yield a secure fixed-length MAC:

- Modify CBC-MAC so that a random IV is used each time a tag is computed (and the IV is output along with t_l). I.e., $t_0 \leftarrow \{0, 1\}^n$ is chosen uniformly at random rather than being fixed to 0^n , and the tag is (t_0, t_l) .
- Modify CBC-MAC so that all blocks t_1, \dots, t_l are output (rather than just t_l).

Problem 2

In this question we define basic notation required for protocol exchange and ask you what vulnerabilities several protocols may possess. We define the following notation.

- We let A, B denote Alice and Bob respectively.
- We let C denote a trusted third-party server, who adheres honestly to the protocol.
- Let Id_A, Id_B, Id_C, Id_E , also denote the publicly known identities of Alice, Bob, the trusted third-party server and Eve, respectively.
- We let E denote Eve, an adversary who has the power to passively eavesdrop, modify, block, store or entirely replace any transmission they perceive.
- We let N_i (for $i \in \mathbb{N}$) denote a *Nonce*, or a randomly sampled number used only once. Any party may create any number of nonces.
- $\{m\}_{k(X,Y)}$ denotes that the message m has been encrypted with a symmetric key k shared by X and Y . m may be recursively defined as $m = m', m''$ or $m = \{m'\}_k$ or $m = Id_x$ or $m = N_i$.
- $n. X \longrightarrow Y : m$ is interpreted as "In step n , X sends Y the message m ".

Alice and Bob live in an almost perfect world, where they have unbreakable symmetric-key encryption. This world (much like yourselves in the course) has not yet seen the introduction of public-key cryptography. The only thing that they both know is the public identity of each other and that C is a trusted third-party server, with whom they share the keys with (A and C both possess knowledge of $k(A, C)$ whilst B and C both possess knowledge of $k(B, C)$).

In protocol 1) Alice and Bob seek to establish a shared secret key $k(A, B)$.

In protocol 2), Bob seeks to confirm that he is communicating with Alice.

Can the protocols be subverted by Eve?

Protocol 1: Key Establishment

- $A \longrightarrow B : Id_A, \{Id_A, N_a\}_{k(A,C)}$
- $B \longrightarrow C : Id_B, \{Id_A, \{Id_A, N_a\}_{k(A,C)}\}_{k(B,C)}$
- $C \longrightarrow B : Id_C, \{Id_A, N_a\}_{k(B,C)}$
- $k(A, B) := N_a$

Protocol 2: Identity establishment

- $A \longrightarrow B : Id_A$
- $B \longrightarrow A : N_b$
- $A \longrightarrow B : \{N_b\}_{k(A,C)}$
- $B \longrightarrow C : \{A, \{N_b\}_{k(A,C)}\}_{k(B,C)}$
- $C \longrightarrow B : \{N_b\}_{k(B,C)}$

Problem 3

Let (Gen_1, H_1) , (Gen_2, H_2) be two hash functions. Define (Gen, H) so that Gen runs Gen_1 and Gen_2 to obtain keys s_1 and s_2 , respectively.

Then define $H^{s_1, s_2}(x) := H_1^{s_1}(x) || H_2^{s_2}(x)$.

- (a) Prove that if at least one of (Gen_1, H_1) and (Gen_2, H_2) is collision resistant, then (Gen, H) is collision resistant.
- (b) Is (Gen, H) pre-image resistant if at least one of (Gen_1, H_1) and (Gen_2, H_2) is pre-image resistant?

Problem 4

The definition of collision-resistance for hash-functions was provided to you in the lectures:

Definition (Collision-resistant hash-function)

A hash-function $\Pi = (\text{Gen}, H)$ is collision resistant if for all probabilistic polynomial-time adversaries \mathcal{A} there is a negligible function negl such that the success probability of \mathcal{A} winning the $\text{Hash-coll}_{\mathcal{A}, \Pi}(n)$ experiment defined below is negligible.

The collision-finding experiment $\text{Hash-coll}_{\mathcal{A}, \Pi}(n)$:

1. A key $s \leftarrow \text{Gen}(1^n)$ is generated.
2. The adversary \mathcal{A} is given s and outputs $x, x' \in \{0, 1\}^*$ (the length is restricted to $x, x' \in \{0, 1\}^{l(n)}$ if Π is a fixed length hash-function).
3. The output of the experiment is defined to be 1 if and only if $x \neq x'$ and $H^s(x) = H^s(x')$. In this case we say that \mathcal{A} has found a collision.

Create a formal definition for *second-preimage resistance* as given in the lectures and prove that a collision-resistant hash-function is also second-preimage resistant.

Problem 5

Let (Gen, H) be a collision-resistant hash function. Is (Gen, \hat{H}) defined by $\hat{H}^s(x) := H^s(H^s(x))$ necessarily collision resistant?

Problem 6

Before HMAC was invented, it was quite common to define a MAC by $\text{Mac}_k(m) = H^s(k || m)$ where H is a collision-resistant hash-function. Show that this is not a secure MAC when H is constructed via the Merkle-Damgård transform.