Part II

Partial differential equations

3. First order semilinear PDEs: Method of characteristics

3.1 Introduction

Consider PDEs for unknown fn $z = z(x,y)$ of form

$$P(x,y)\,\partial_x z(x,y) + Q(x,y)\,\partial_y z(x,y) = R(x,y,z(x,y))$$

(PDE)

Goal:

Determine solutions of (PDE) given some data (prescribed z on some curve $z(x,y) = g(x,y)$ for all $(x,y) \in$ 1D set)

& understand WHERE the solution is uniquely determined by $\left\{ \begin{array}{l} \text{(PDE)} \\ + \\ \text{data} \end{array} \right.$ "domain of definition"

Idea:

- Consider solution surface $\Sigma = \{ (x,y,z(x,y)) : (x,y) \in \text{domain of def.} \}$

1. Σ as a parametrised surface
2. Solve for $z(x,y)$.

Rough idea:

Having $z = z(x, y)$ prescribed on a curve, say parametrised by $(\gamma_1(s), \gamma_2(s))$

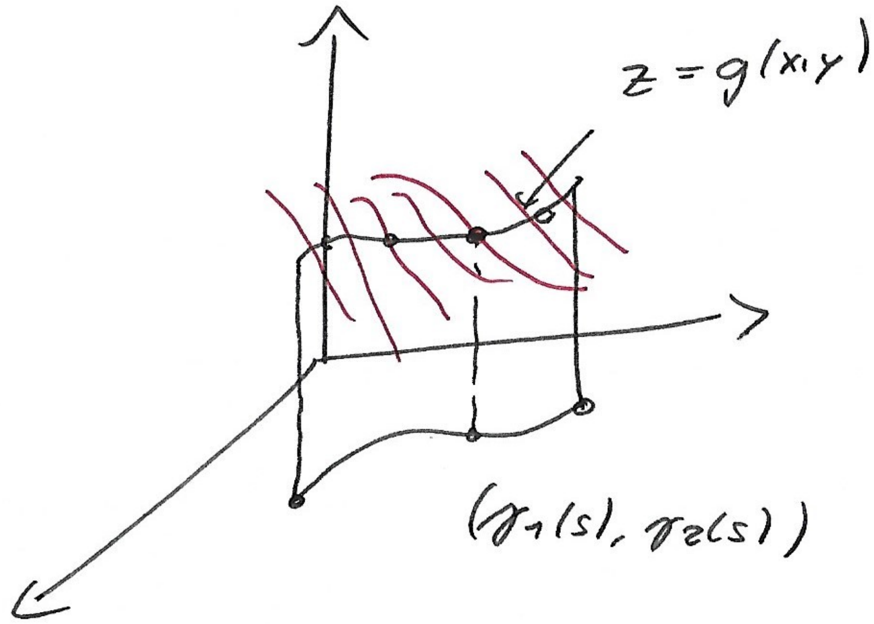
we can view prescribed data as asking that curve

$$(\gamma_1(s), \gamma_2(s), \gamma_3(s))$$

$$\gamma_3(s) = g(\gamma_1(s), \gamma_2(s))$$

is contained in solution surface.

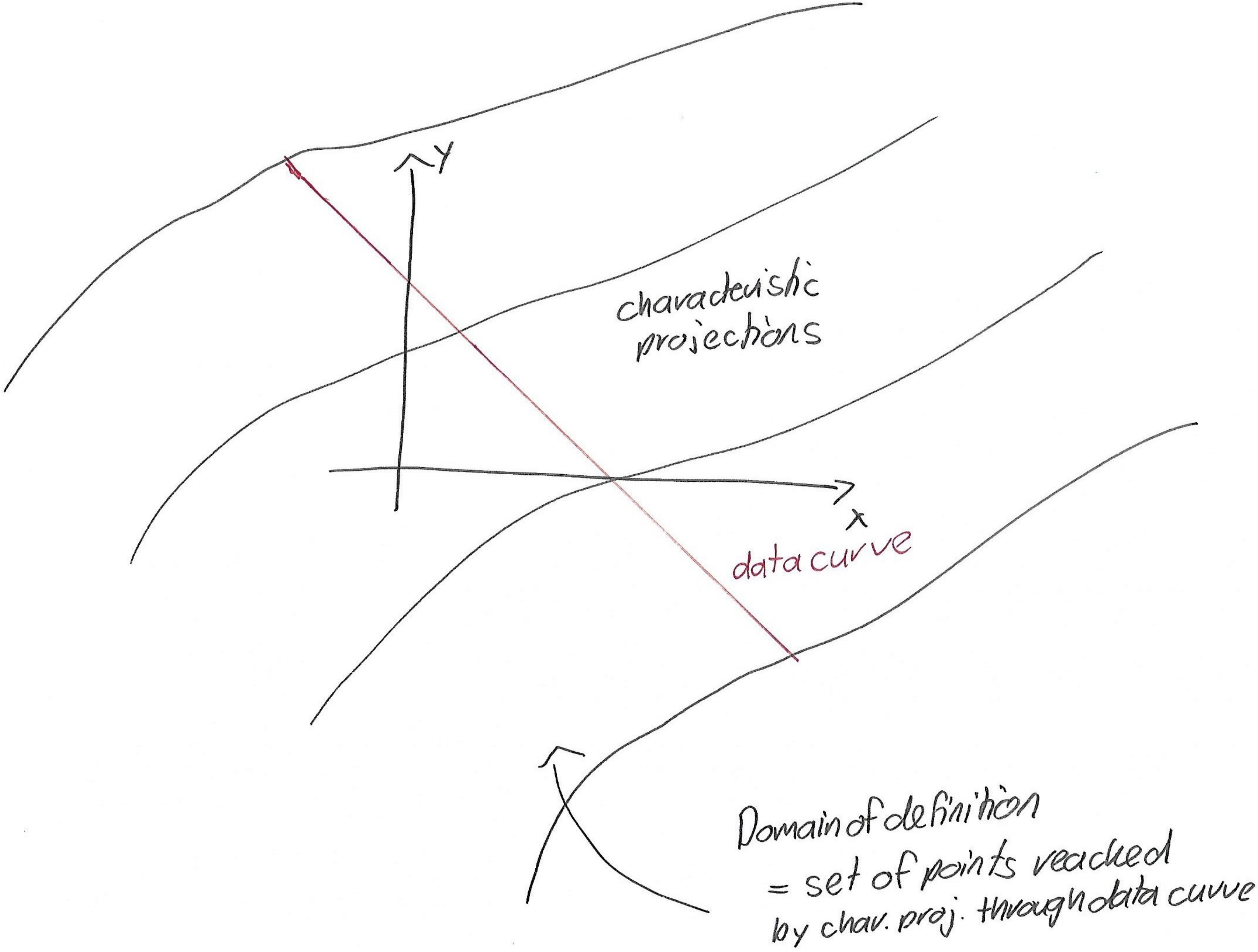
We then want to determine special curves = "characteristics" which have the property that if they start on Σ then the



whole curve = characteristic is in the solution surface.

→ 1D "set" 1 parameter family of 1D curves

→ 2D surface.


y
x
characteristic projections
data curve
Domain of definition
= set of points reached
by char. proj. through data curve