3.4 Examples:

Example 2 (similar to ex. (b)) → notes.

$$\begin{cases} x\,\partial_x z + y\,\partial_y z = z \\ z = 1 \text{ on } \{(x,y) : (x-2)^2 + y^2 = 2,\ x \geq 0\} \end{cases}$$

Characteristics $\begin{pmatrix} x \\ y \\ z \end{pmatrix}(t) = \begin{pmatrix} A \\ B \\ C \end{pmatrix} \cdot e^t$

$\parallel$

half lines through $(0,0,0)$ in $\mathbb{R}^3$

char. proj.: half-lines in $\mathbb{R}^2$

$$\gamma(s) = (2 - \sqrt{2}\cdot\cos s,\ \sqrt{2}\sin s,\ 1)$$

$$s \in [0, \pi]$$

$x(t,s) = (2 - \sqrt{2}\cos s)\cdot e^t$

$y(t,s) = \sqrt{2}\sin s \cdot e^t$

$z(t,s) = 1 \cdot e^t$ $\qquad t \in \mathbb{R}$, $s \in [0, \pi]$

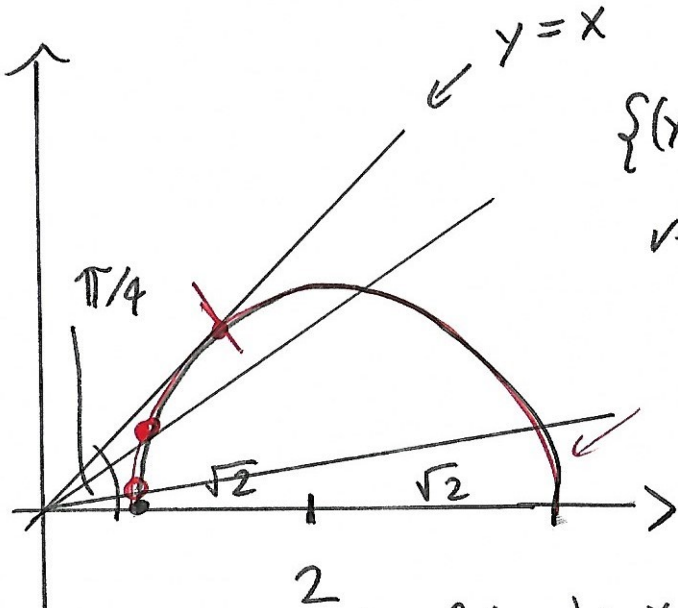
Cannot solve $x = x(s,t)$, $y = y(s,t)$ uniquely for s,t in this range.

$s \in [0, \pi/4) \to$ 1 solution to "restricted problem"

$s \in (\pi/4, \pi) \to$ different sol.

$z = 1$ on $\{(x,y)(s) : s \in [0, \frac{\pi}{4})\}$

resp. $\{(x,y)(s),\ s \in (\frac{\pi}{4}, \pi]\}$ $z = 1$

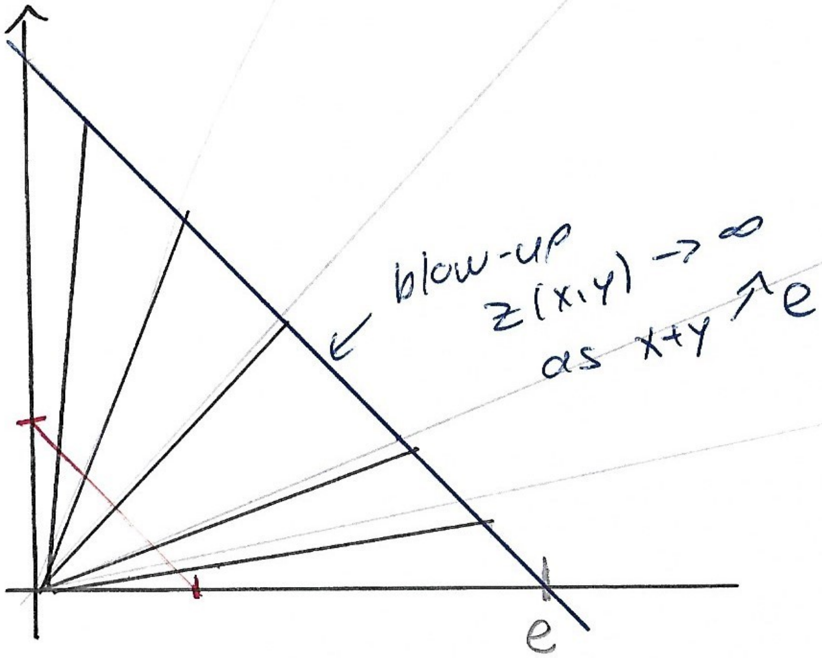


Domain of def. $= \{(x,y) : x \geq 0,\ 0 \leq y \leq x\}$.

Example 3

$$x\,\partial_x z + y\,\partial_y z = z^2$$

$$z = 1 \quad \text{on} \quad y = 1-x\,,\; x \in [0,1]$$



$$(x,y)(t) = (A,B)\cdot e^t$$

$$\dot{z} = z^2 \quad \to -\frac{1}{z} = \int \frac{dz}{z^2} = \int dt = t + c$$

$$\to z = \frac{1}{-c-t} = \frac{1}{1-t}$$

initial curve

$$(s,\, 1-s,\, 1) \qquad s \in [0,1]$$

So:

$$x(t,s) = s\,e^t$$
$$y(t,s) = (1-s)\,e^t \qquad s \in [0,1]$$
$$z(t,s) = \frac{1}{1-t} \qquad t \in (-\infty, 1)$$

Char. proj.

$$\binom{s}{1-s} e^t \,, \quad t \in (-\infty, 1)$$

Domain of def

$$\Omega = \{(x,y) : x, y \geq 0,\; x+y \leq e\}$$

Note: $e^t = x+y \;\to\; t = \log(x+y)$

$$z(x,y) = \frac{1}{1-\log(x+y)} : (x,y) \in \Omega .$$