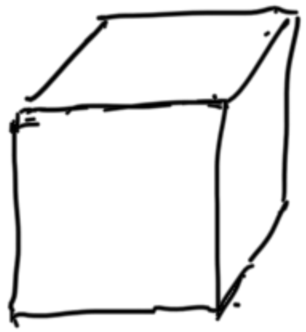
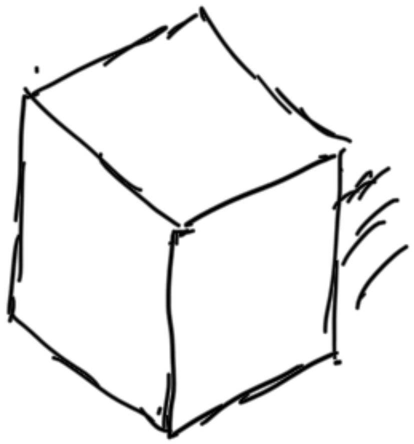
PROJECTIVE GEOMETRY LECTURE 1

How to draw a cube?

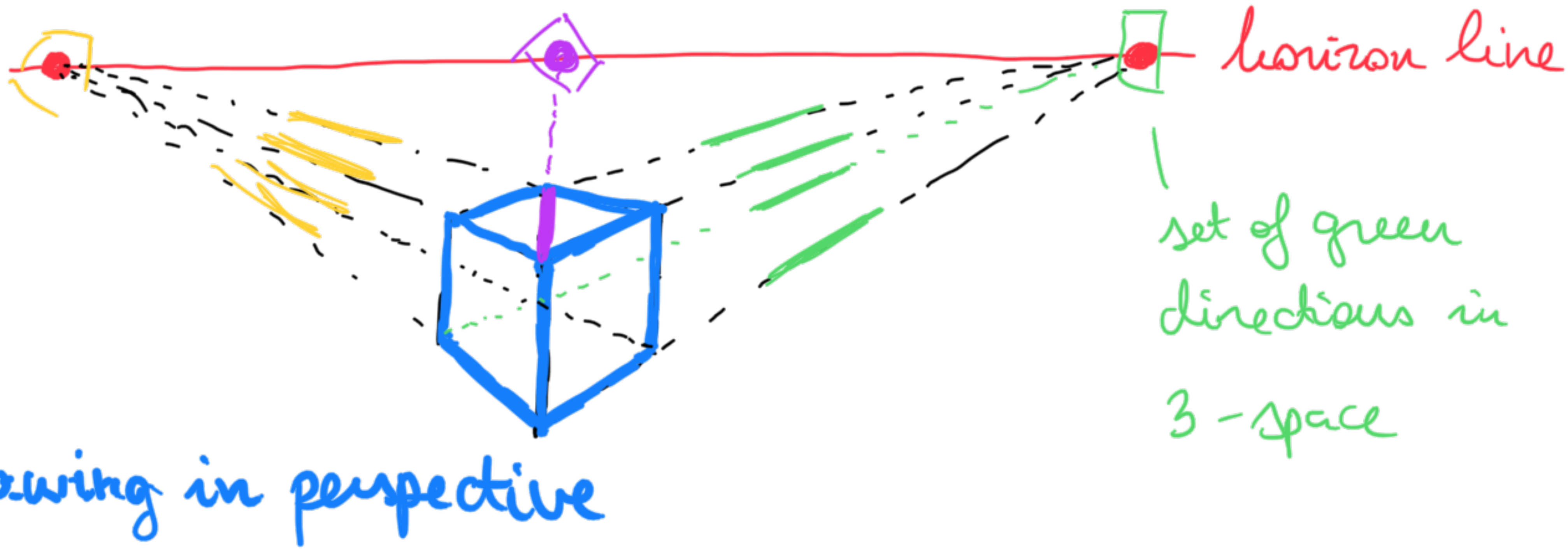
i.)



ii.)



iii.)

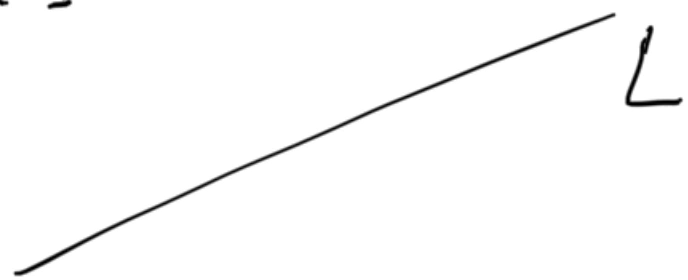


Key messages

- Two sets of dotted lines (leaning left/right) represent parallel lines in 3-space! ideal points
  These parallel directions (in 3-space) meet on the horizon line in a common point.
- These special points themselves form the line of horizon
  - ideal line
  - line $\boxed{\text{at infinity}}$

Recall: linear geometry of real plane $\mathbb{R}^2$

- points p.
- lines



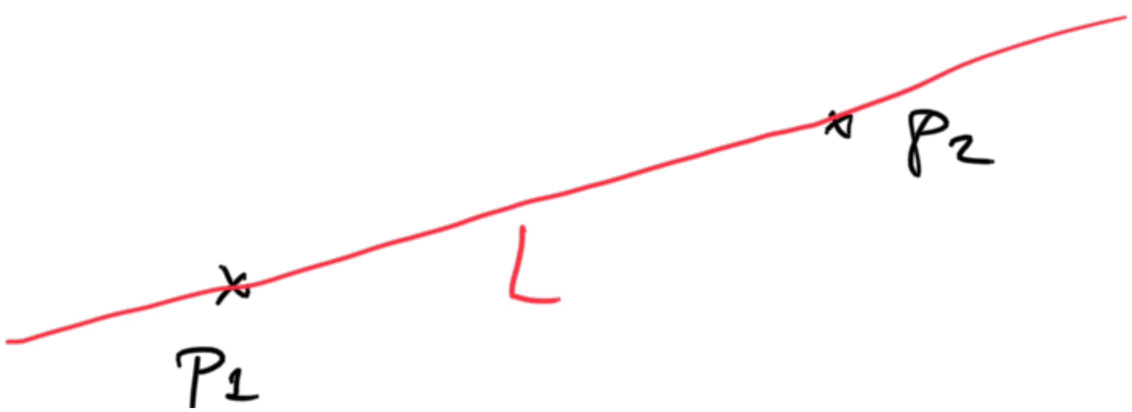
$p \notin L$

- incidence / meet / contain relation:

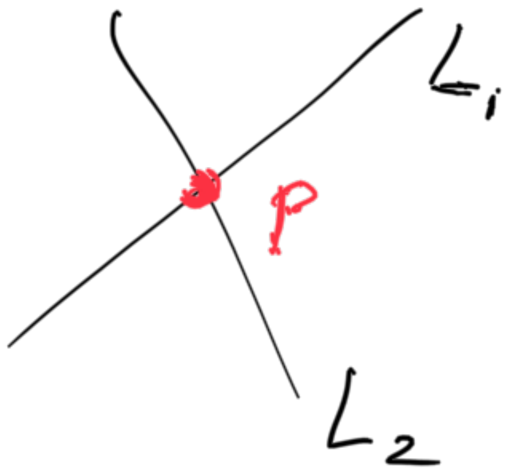
$$p \in L \quad \text{or} \quad p \notin L$$

$q \in L$

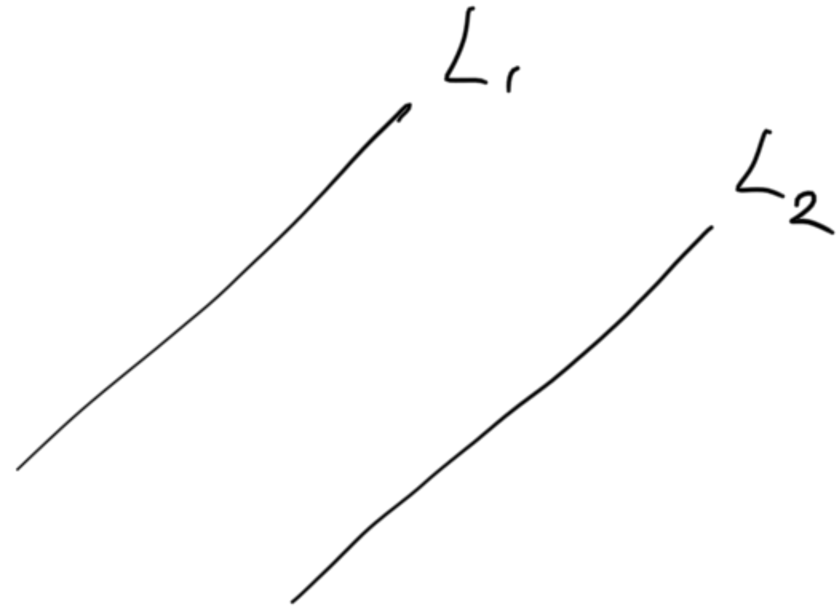
Property 1 For every two distinct points, $p_1 \neq p_2$, there exists a unique line L "containing both" $p_1 \in L$, $p_2 \in L$.



__Property 2__ For every two distinct lines $L_1 \neq L_2$, there exists __at most__ one point "of intersection" i.e. p such that $p \in L_1$ and $p \in L_2$.



intersecting lines in $\mathbb{R}^2$

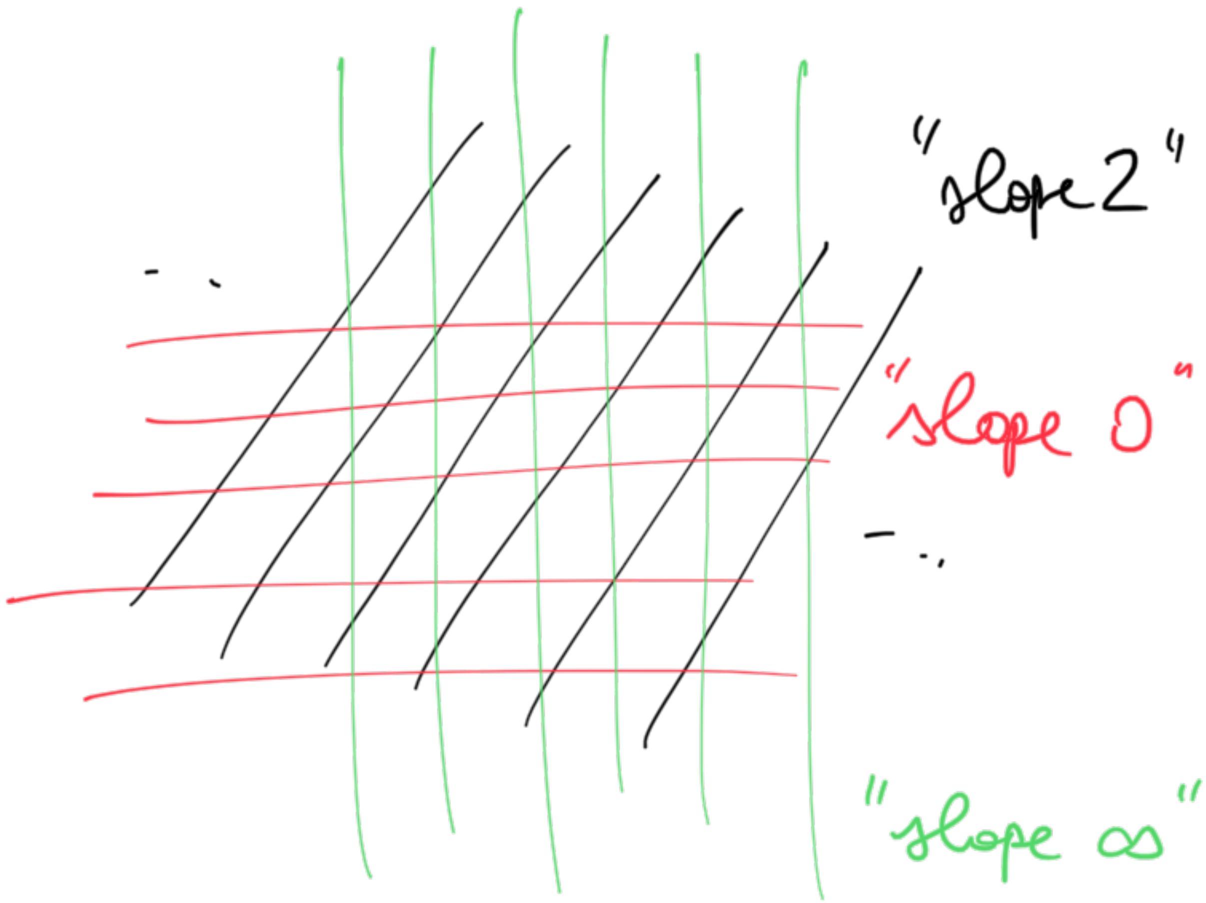


parallel lines in $\mathbb{R}^2$

$L_1 \parallel L_2$

Being parallel is an equivalence relation on lines in $\mathbb{R}^2$

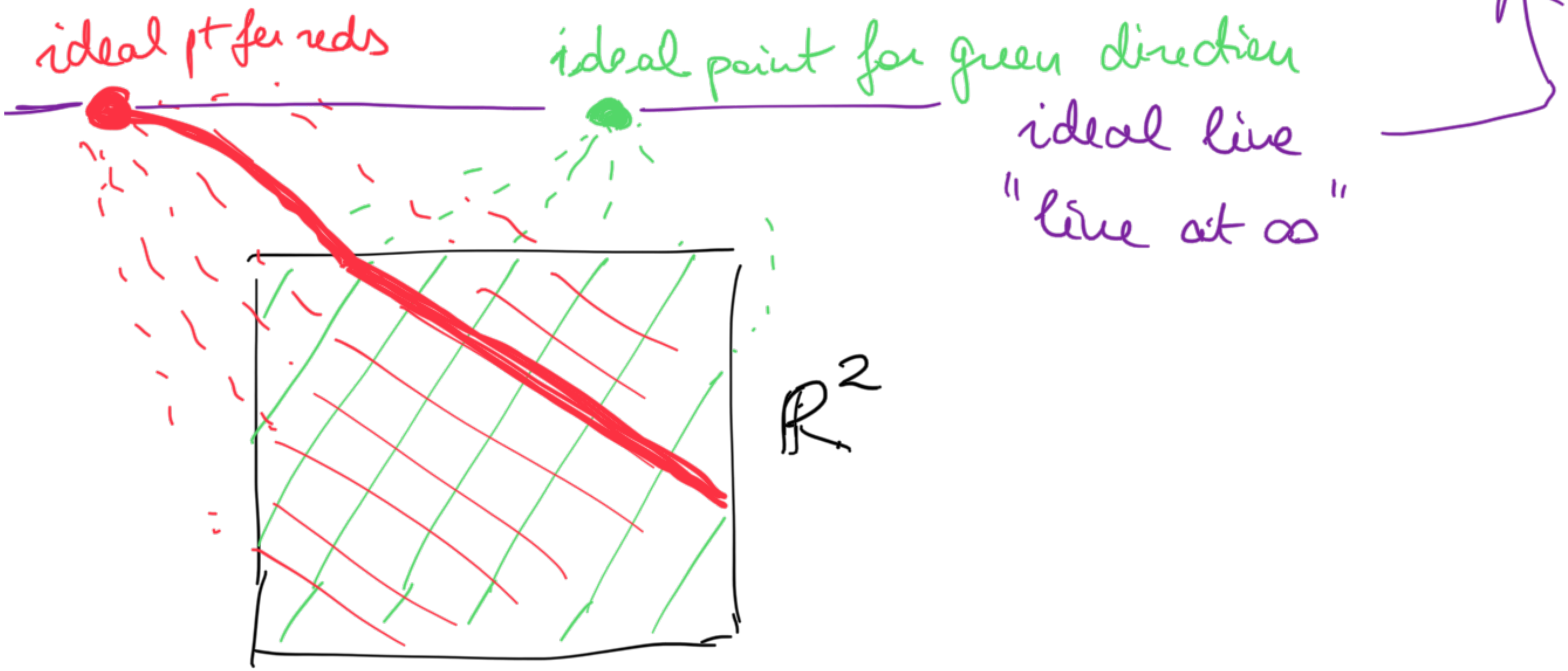
Equivalence classes: __directions__ / lines of "same slope"



What is real projective plane?

Slogan: "parallel lines meet at infinity"

$$\mathbb{R}\mathbb{P}^2 = \mathbb{R}^2 \amalg \{\text{one "ideal point" for each direction}\}$$



Ordinary lines in $\mathbb{R}^2$ are extended to projective lines by including their ideal pt.

proj line = ordinary line $\amalg$ ideal pt

one pt

Check in this geometry, Properties 1 & 2 hold *with no exceptions*!

Proj. Prop. 1 Any two different projective (ordinary or ideal) pts span a unique proj line

Proj. Prop 2 Any two different projective lines meet in a unique projective point!

We call original $\mathbb{R}^2$ the *affine part* of $\mathbb{RP}^2$

$$\text{projective space} = \underset{\text{finite}}{\text{affine space}} \sqcup \underset{\text{“at } \infty \text{”}}{\text{ideal points}}$$

Two major generalizations

a.) $\mathbb{R}P^2 \longrightarrow \mathbb{R}P^n = \mathbb{R}^n \amalg (\text{ideal points})$

b.) $\mathbb{R} \longrightarrow$ arbitrary field $\mathbb{F}$ $\quad \mathbb{F}P^n$