BO1 History of Mathematics
Lecture II
Dissemination and development
(AD 500 – AD 1600)
Part 2: Napier's invention of logarithms

MT 2020 Week 1

A case study of a text from 1614

Napier's invention of logarithms:

- what did 17th-century mathematics look like?
- how can we begin to read historical texts?

Napier's definition of a logarithm (of a sine)

The **Logarithme** *therefore of any sine is a number very neerely expressing the line, which increased equally in the meane time, whiles the line of the whole sine decreased proportionally into that sine, both motions being equal-timed, and the beginning equally swift.*

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Context:	who? when? where? why?
Content:	what is it about? how is it written?
Significance:	why did/does it matter?

Context — who?

John Napier (1550–1617), Merchiston, Scotland

Scottish landowner with interests in:

- mining
- calculating aids
- astrology/astronomy
- The Revelation of St John

See *Oxford Dictionary of National Biography*: http://www.oxforddnb.com/view/article/19758

Context — why?

From Napier's preface to the translation of 1616:

> *Seeing there is nothing (right well-beloved Students of the Mathematics) that is so troublesome to mathematical practice, nor that doth more molest and hinder calculators, than the multiplications, divisions, square and cubical extractions of great numbers, which besides the tedious expense of time are for the most part subject to many slippery errors, I began therefore to consider in my mind by what certain and ready art I might remove those hindrances.*

Context — why?

Inspired by the 16th-century technique of prosthaphaeresis:

the use of trigonometric identities such as

$$\cos x \cos y = \frac{1}{2}\left[\cos(x+y) + \cos(x-y)\right]$$
$$\sin x \sin y = \frac{1}{2}\left[\cos(x-y) - \cos(x+y)\right]$$

to convert multiplication into addition.

Context — in what form, and in which language?

Original Latin text of 1614:

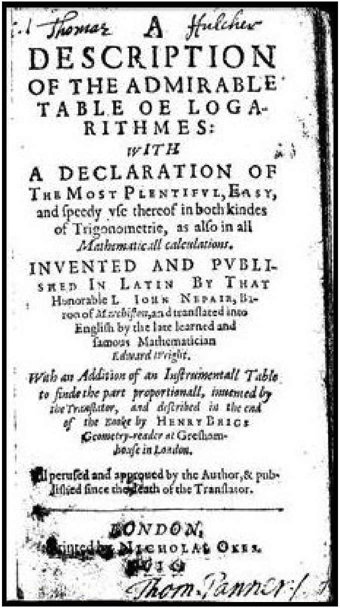
Mirifici logarithmorum canonis descriptio

translated into English by Edward Wright in 1616 as

A description of the admirable table of logarithms

Scanned text available via SOLO

Napier's 1616 title-page decoded



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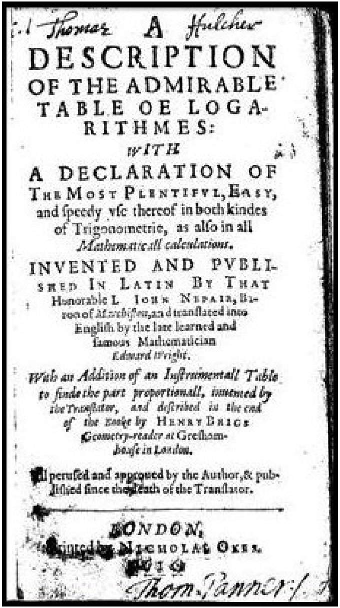
Thomas A Hulcher
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Thom. Panner.

Inventor:
: John Napier (1550–1617)

Translator:
: Edward Wright (?1558–1615) (interests: navigation, charts and tables)

Additional material:
: Henry Briggs (1561–1630) Gresham Professor of Geometry, later Savilian Professor of Geometry at Oxford (interests: navigation)

Printer:
: Nicholas Okes

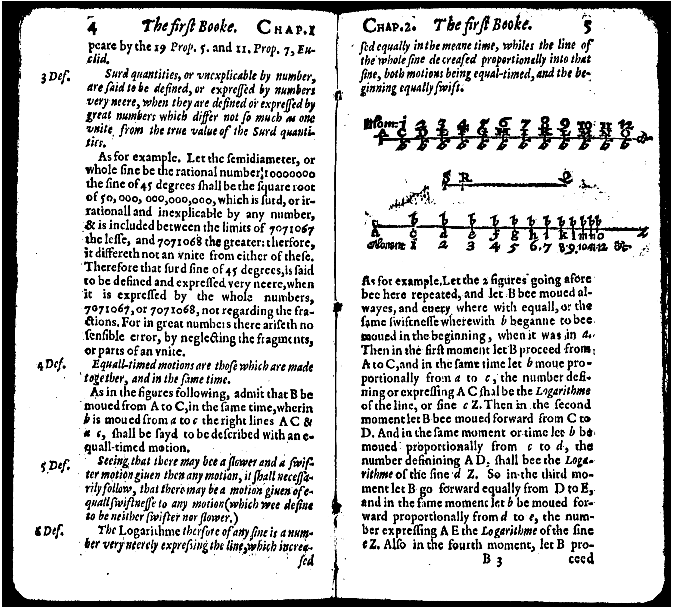
Readers:
: Thomas Hulcher,
: Thomas Panner

Napier's logarithms: content

Recall:

> *The* **Logarithme** *therefore of any sine is a number very neerely expressing the line, which increased equally in the meane time, whiles the line of the whole sine decreased proportionally into that sine, both motions being equal-timed, and the beginning equally swift.*

Napier's logarithms



4 *The firſt Booke.* CHAP. I

peare by the 19 *Prop.* 5. and 11. *Prop.* 7, *Euclid.*

3 *Def.* *Surd quantities, or vnexplicable by number, are ſaid to be defined, or expreſſed by numbers very neere, when they are defined or expreſſed by great numbers which differ not ſo much as one vnite from the true value of the Surd quantities.*

As for example. Let the ſemidiameter, or whole ſine be the rational number, 10000000 the ſine of 45 degrees ſhall be the ſquare root of 50, 000, 000,000,000, which is ſurd, or irrationall and inexplicable by any number, & is included between the limits of 7071067 the leſſe, and 7071068 the greater: therfore, it differeth not an vnite from either of theſe. Therefore that ſurd ſine of 45 degrees, is ſaid to be defined and expreſſed very neere, when it is expreſſed by the whole numbers, 7071067, or 7071068, not regarding the fractions. For in great numbers there ariſeth no ſenſible error, by neglecting the fragments, or parts of an vnite.

4 *Def.* *Equall-timed motions are thoſe which are made together, and in the ſame time.*

As in the figures following, admit that B be moued from A to C, in the ſame time, wherin *b* is moued from *a* to *c* the right lines A C & *a c*, ſhall be ſayd to be deſcribed with an equall-timed motion.

5 *Def.* *Seeing that there may bee a ſlower and a ſwifter motion giuen then any motion, it ſhall neceſſarily follow, that there may be a motion giuen of equall ſwiftneſſe to any motion (which wee define to be neither ſwifter nor ſlower.)*

6 *Def.* *The* Logarithme *therfore of any ſine is a number very neerely expreſing the line, which increaſed*

CHAP. 2. *The firſt Booke.* 5

ſed equally in the meane time, whiles the line of the whole ſine decreaſed proportionally into that ſine, both motions being equal-timed, and the beginning equally ſwift.

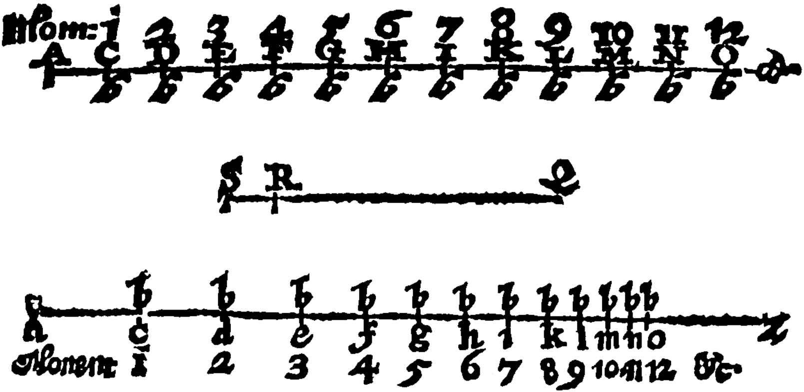
Mom: 1 2 3 4 5 6 7 8 9 10 11 12

Moment 1 2 3 4 5 6.7 8.9.10.11.12 &c.

As for example. Let the 2 figures going afore bee here repeated, and let B bee moued alwayes, and euery where with equall, or the ſame ſwiftneſſe wherewith *b* beganne to bee moued in the beginning, when it was in *a*. Then in the firſt moment let B proceed from A to C, and in the ſame time let *b* moue proportionally from *a* to *c*, the number defining or expreſſing A C ſhal be the *Logarithme* of the line, or ſine *c* Z. Then in the ſecond moment let B bee moued forward from C to D. And in the ſame moment or time let *b* be moued proportionally from *c* to *d*, the number defining A D, ſhall bee the *Logarithme* of the ſine *d* Z. So in the third moment let B go forward equally from D to E, and in the ſame moment let *b* be moued forward proportionally from *d* to *e*, the number expreſſing A E the *Logarithme* of the ſine *e* Z. Alſo in the fourth moment, let B proceed

B 3

Napier's logarithms



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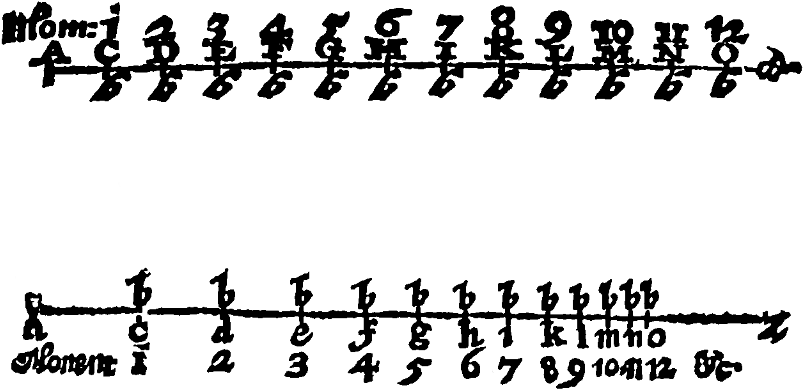

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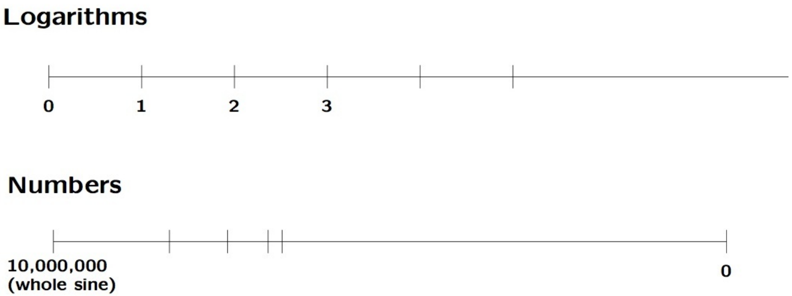

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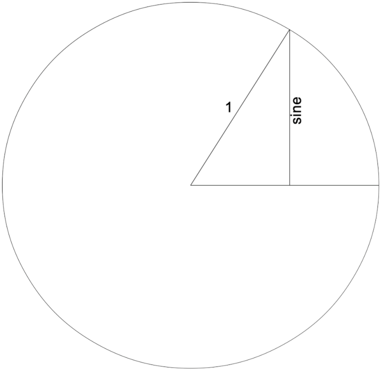

Sine of angle at centre varies between 0 and ± 1 as the labelled radius sweeps around the circle

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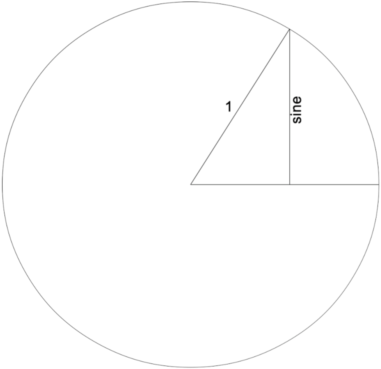

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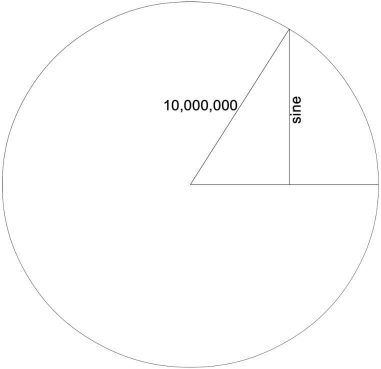
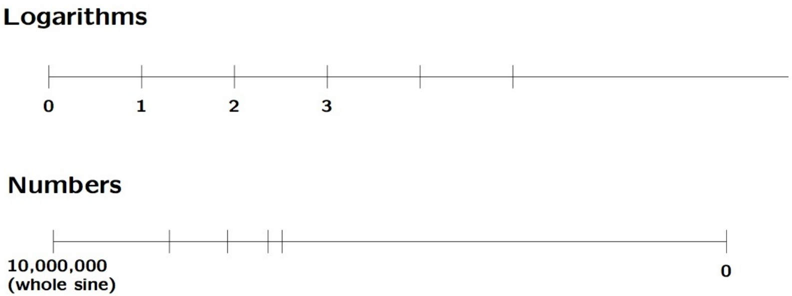


Sine of angle at centre varies between 0 and $\pm 10,000,000$ as the labelled radius sweeps around the circle

Napier's logarithms



Napier's logarithms (1614)

In modern terms (i.e., not Napier's):

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No notion of base, although Nap log 'nearly' has base $\frac{1}{e}$ — see: Robin Wilson, *Euler's Pioneering Equation*, OUP, 2019, p. 101

Modifications by Napier and Briggs (1617)

Definition revised to remove the need to subtract $\text{Nap}\log 1$

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Briggs produced *Logarithmorum chilias prima* (*The first thousand logarithms*) in 1617, followed by his *Arithmetica logarithmica* in 1624, which contained logarithms of 1 to 20,000 and 90,000 to 100,000, all to 14 decimal places (calculated by hand); the gap in the table was filled by Adriaan Vlacq in 1628

Napier's logarithms

One last time:

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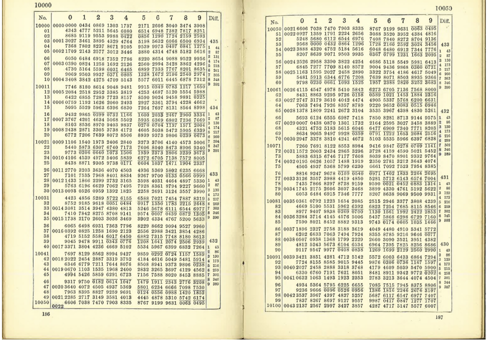
Significance



Napier's logarithms:

- caught on very quickly
- a calculating aid (until the 1980s)
- logarithms rapidly came to have other interpretations (as you know, and as we shall see)