BO1 History of Mathematics
Lecture VI
Successes of and difficulties with the calculus: the 18th-century beginnings of 'rigour'
Part 1: Publication, acceptance, and successes

MT 2020 Week 3

Summary

Part 1

- Publication and acceptance of the calculus
- Some successes of the calculus

Part 2

- Functions

Part 3

- Problems with the calculus
- Some responses: beginnings of ‘rigour’ in Analysis

Reminder: a comparison from lecture IV

Newton (1664–65):	Leibniz (1673–76):
rules for quadrature	rules for quadrature
rules for tangents	rules for tangents
'fundamental theorem'	'fundamental theorem'
dot notation	'modern' notation
physical intuition:	algebraic intuition
rates of change	rules and procedures
PROBLEM:	PROBLEM:
vanishing quantities o	vanishing quantities du, dv, ...

Newton's publication (or not) of his calculus

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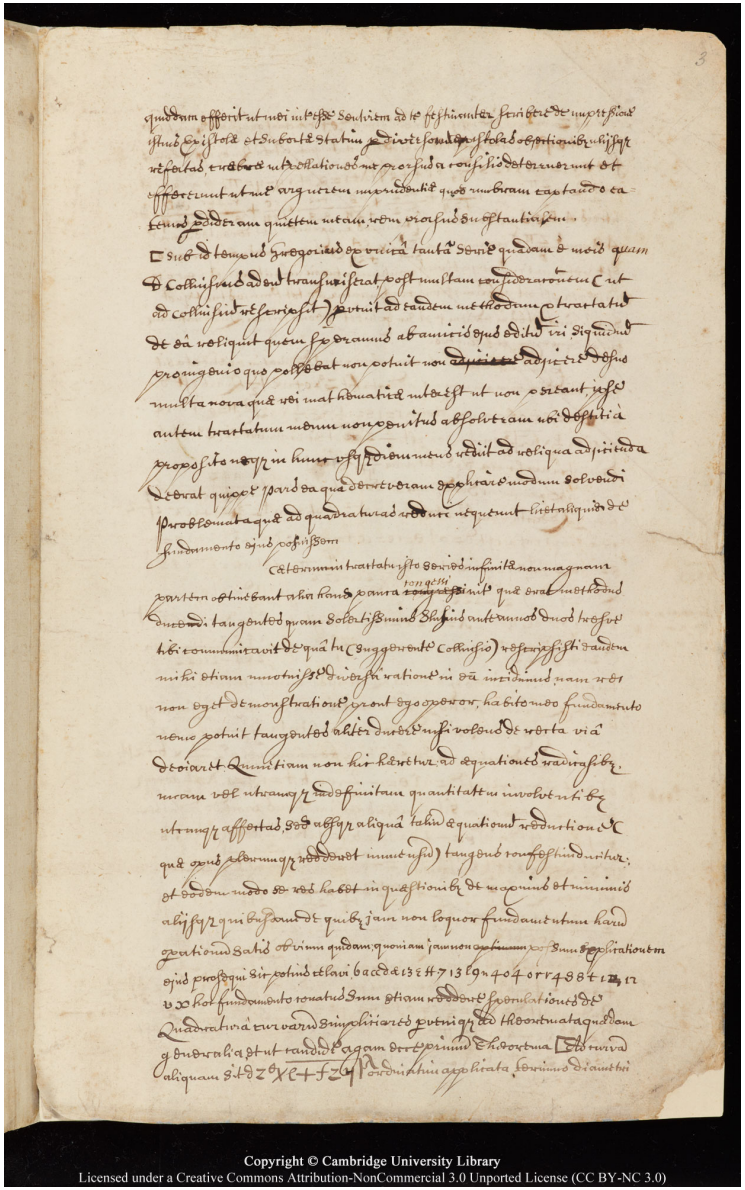
1704: 'Treatise of quadrature' appended to published *Opticks*

Newton's coded message



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Letter from Isaac Newton to Henry Oldenburg, 24 October 1676 ('Epistola posterior')

"The foundation of these operations is evident enough, in fact; but because I cannot proceed with the explanation of it now, I have preferred to conceal it thus: 6*accdae*13*eff*7*i*3*l*9*n*4*o*4*qrr*4*s*8*t*12*vx*."

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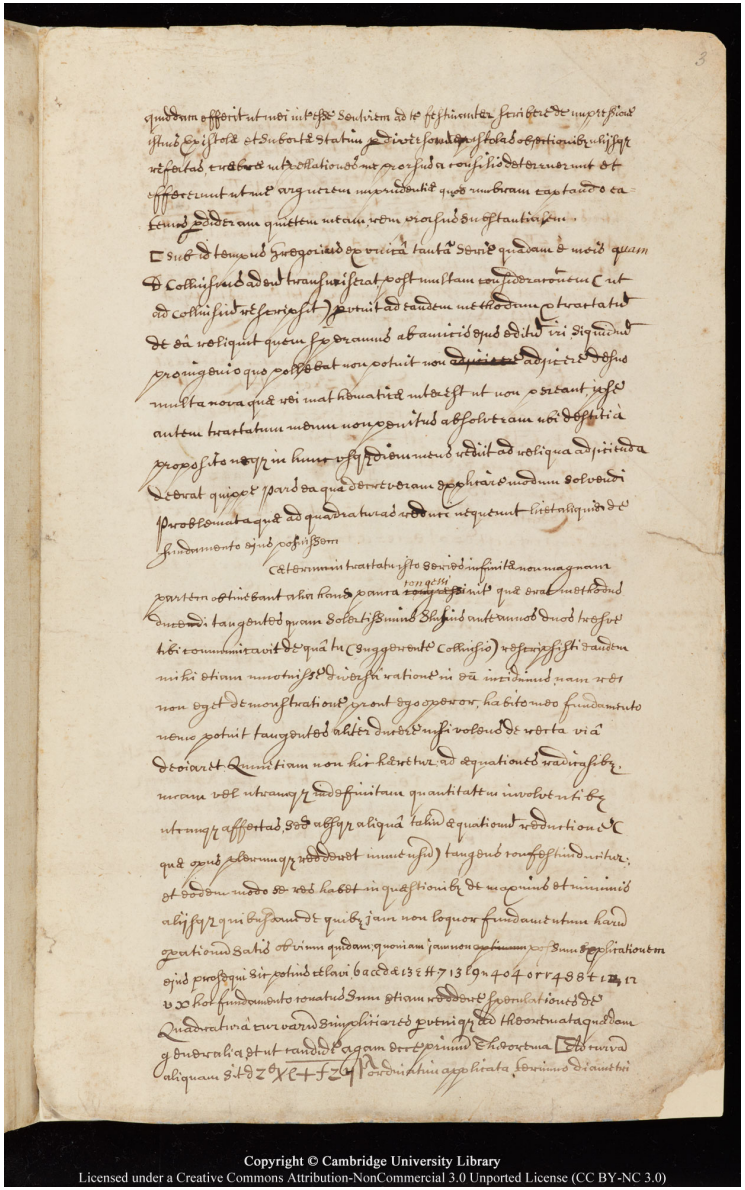

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"Data aequatione quotcunque fluentes quantitates involvente, fluxiones invenire: et vice versa."

= "Given an equation involving any number of fluent quantities, to find the fluxions: and vice versa."

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1696: Exposition by L'Hôpital based on teachings of Johann Bernoulli

Challenge problems

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1687: Isochrone — curve of uniform descent
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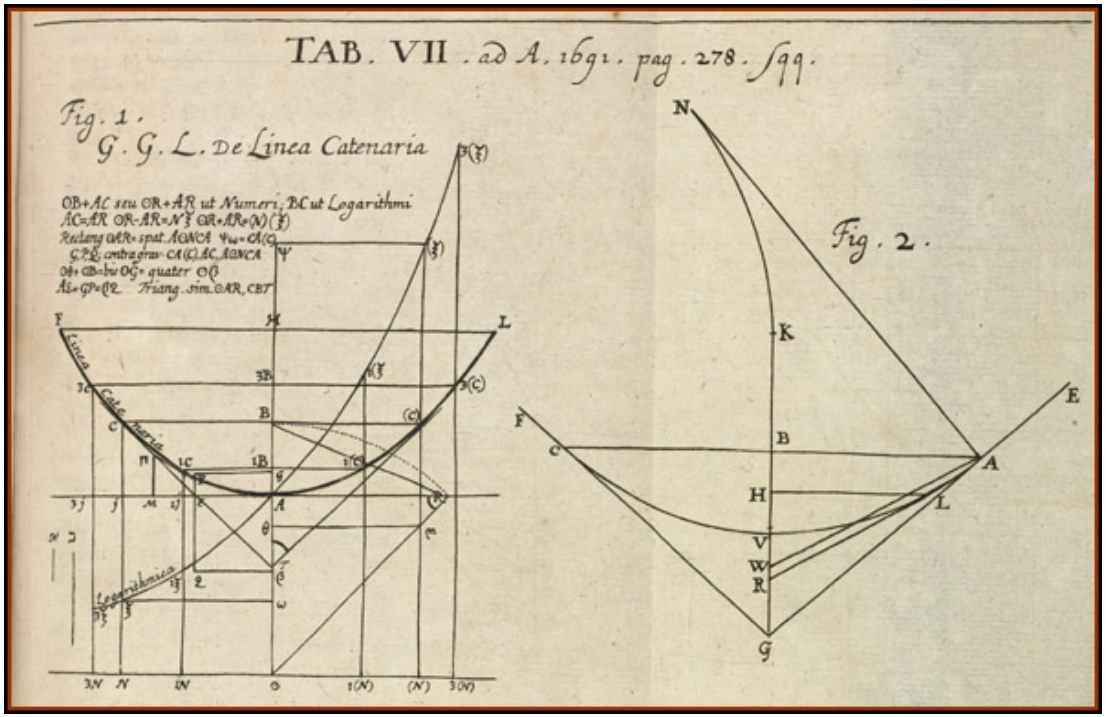
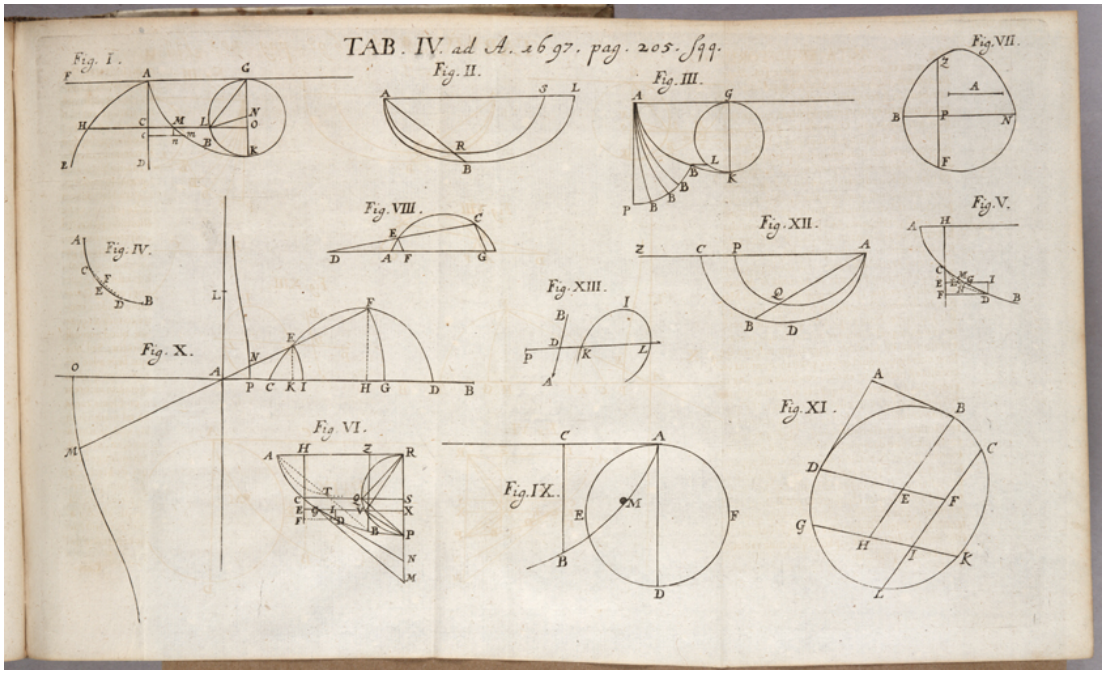
1691: Catenary — curve of a hanging chain
(posed by Jacob Bernoulli; solved by Johann Bernoulli, Huygens, Leibniz)



Leibniz' and Huygens' solutions, *Acta eruditorum*, 1691.

Challenge problems



Solutions by Johann & Jacob Bernoulli, l'Hospital, and Newton, *Acta eruditorum*, 1696.

1696: Brachistochrone — curve of fastest descent (posed by Johann Bernoulli; shown to be cycloid by Jacob Bernoulli, Leibniz, Newton, l'Hôpital)

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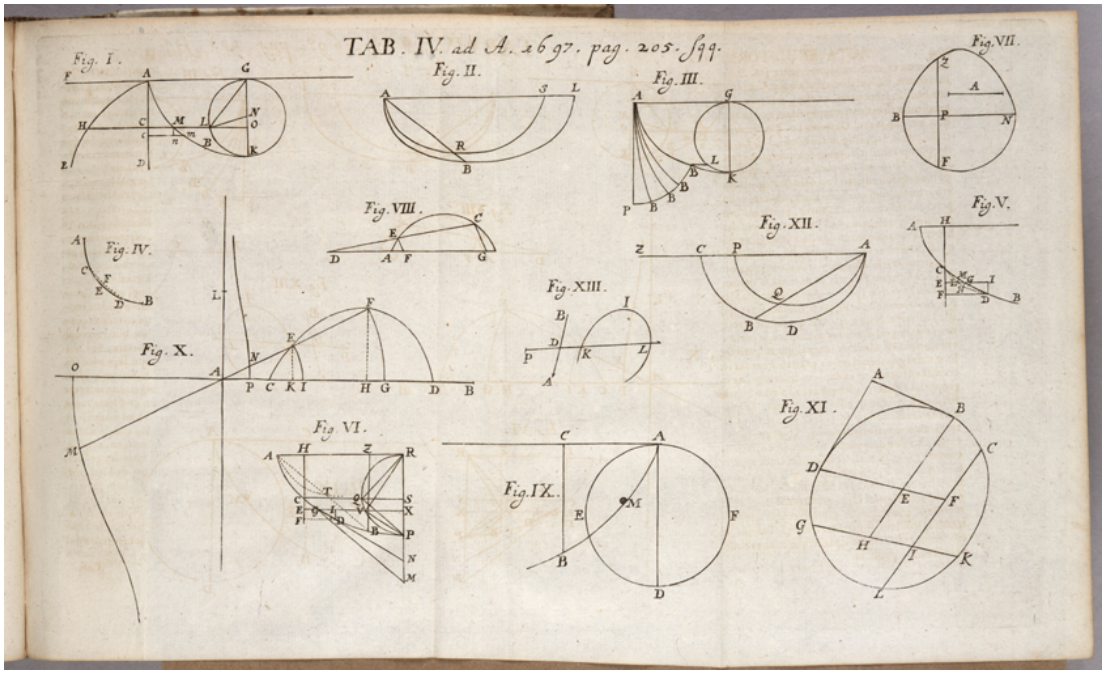

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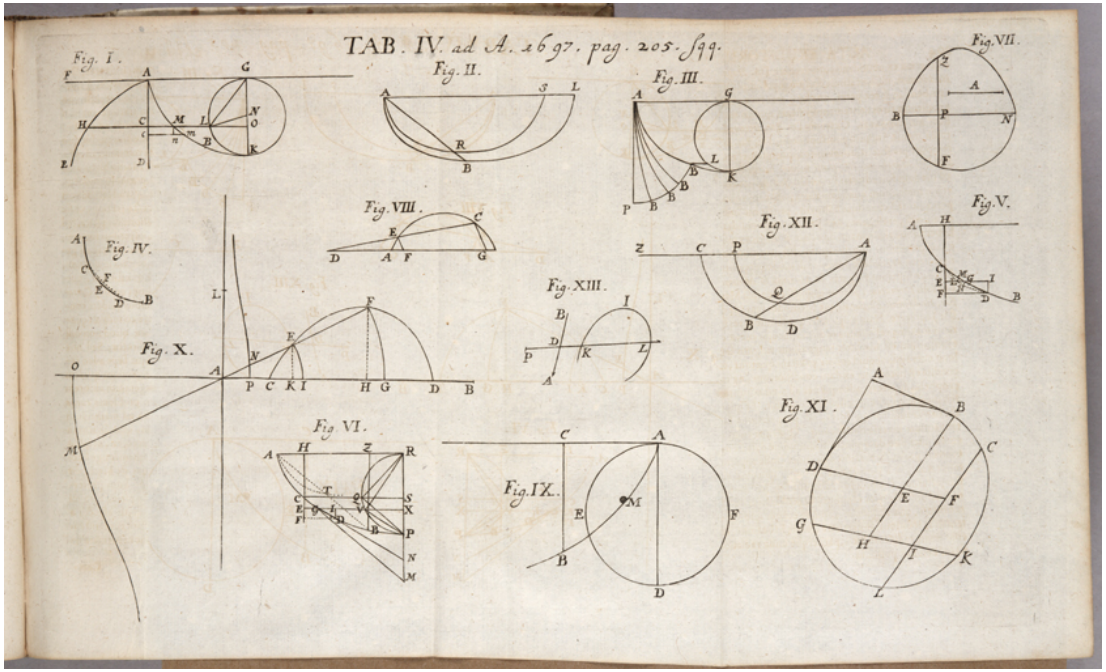

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And many others

People and connections

Christiaan Huygens (1629–1695)

tutored (1670s)

Gottfried Wilhelm von Leibniz (1646–1716)

corresponded (1690s)

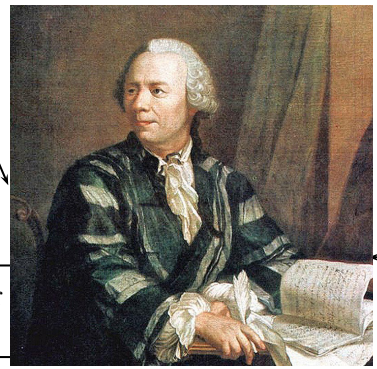
Johann Bernoulli (1667–1748)

Jacob Bernoulli (1655–1705)

tutored (1690s)

Guillaume, Marquis de l'Hôpital (1661–1704)

tutored (1720s)



Leonhard Euler (1707–1783)

collaborated (1730s)

Daniel Bernoulli (1700–1782)

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- consolidate and validate Leibnizian calculus
- introduce new questions about ‘functions’, ‘differentiability’, ‘continuity’, ...