BO1 History of Mathematics
Lecture VI
Successes of and difficulties with the calculus: the 18th-century beginnings of ‘rigour’
Part 3: Difficulties and responses

MT 2020 Week 3

More problems: infinitely small quantities

Thomas Hobbes, *Six lessons to the Professors of Mathematicks* (1656):

> *The least Altitude is Somewhat or Nothing. If Somewhat, then the first character of your Arithmeticall Progression must not be zero;*
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> *If Nothing, then your whole figure is without Altitude and consequently your Understanding nought.*

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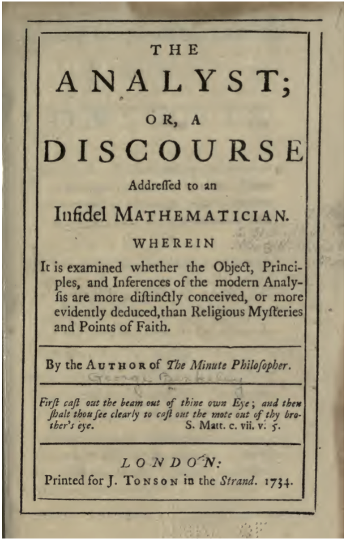
Wallis tried to provide further explanation in his *Due correction for Mr. Hobbes* (1656), but wasn't too concerned by the problems

More problems: infinitely small quantities

George Berkeley (1734)



THE

ANALYST;

OR, A

DISCOURSE

Addreſſed to an

Infidel MATHEMATICIAN.

WHEREIN

It is examined whether the Object, Principles, and Inferences of the modern Analyſis are more diſtinctly conceived, or more evidently deduced, than Religious Myſteries and Points of Faith.

By the AUTHOR of *The Minute Philoſopher*.

Firſt caſt out the beam out of thine own Eye; and then ſhalt thou ſee clearly to caſt out the mote out of thy brother's eye. S. Matt. c. vii. v. 5.

LONDON:

Printed for J. TONSON in the *Strand*. 1734.

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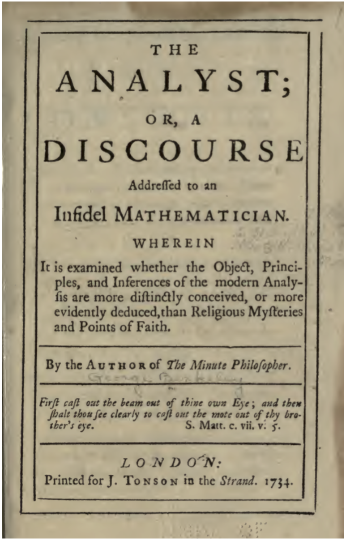

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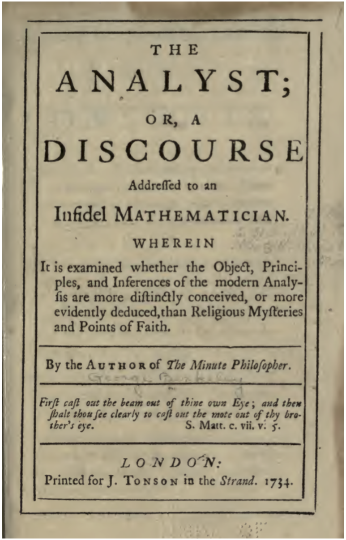
Qu. 43: Whether an algebraist, fluxionist, geometrician, or demonstrator of any kind can expect indulgence for obscure principles or incorrect reasoning? And whether an algebraical note or species can at the end of a process be interpreted in a sense which could not have been substituted for it at the beginning?

More problems: infinitely small quantities



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Qu. 45: Whether, although geometry be a science, and algebra allowed to be a science, and the analytical a most excellent method, in the application nevertheless of the analysis to geometry, men may not have admitted false principles and wrong methods of reasoning?

Some responses to the difficulties

Guillaume Marquis de l'Hôpital, *Analyse des infiniment petits* (1696)

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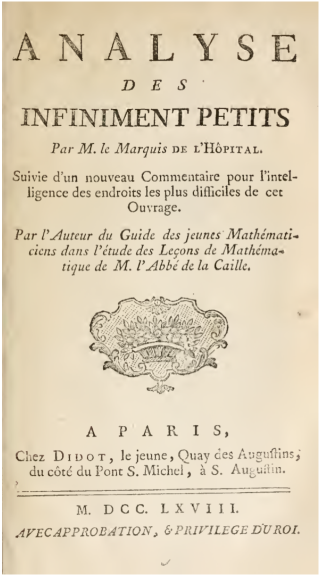
Responses to the difficulties: l'Hôpital



ANALYSE
DES
INFINIMENT PETITS

Par M. le Marquis DE L'HÔPITAL.

Suivie d'un nouveau Commentaire pour l'intelligence des endroits les plus difficiles de cet Ouvrage.

Par l'Auteur du Guide des jeunes Mathématiciens dans l'étude des Leçons de Mathématique de M. l'Abbé de la Caille.

A PARIS,

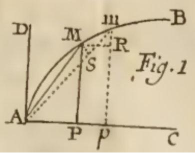
Chez DIDOT, le jeune, Quay des Auguſtins; du côté du Pont S. Michel, à S. Auguſtin.

M. DCC. LXVIII.

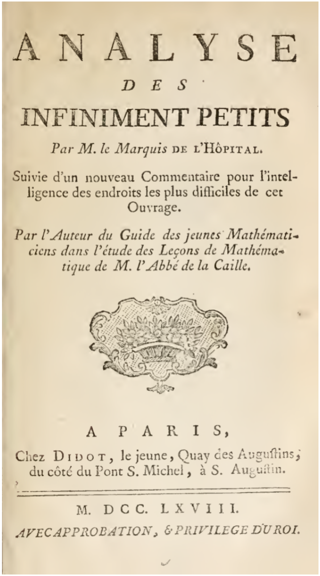
AVEC APPROBATION, & PRIVILEGE DU ROI.

Guillaume, Marquis de l'Hôpital (1696)

Definition. The infinitely small part whereby a variable quantity is continually increased or decreased, is called the differential of that quantity.



Responses to the difficulties: l'Hôpital



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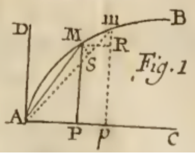
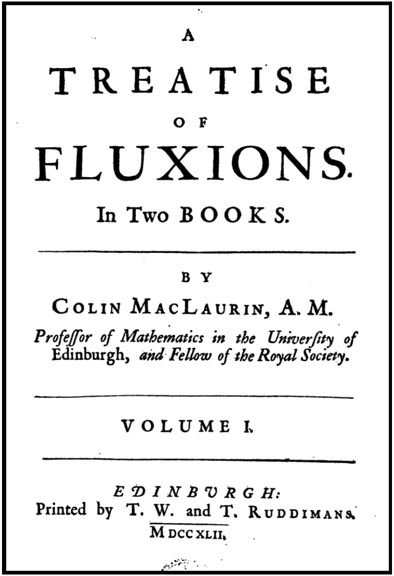


Postulate. Grant that two quantities whose difference is an infinitely small quantity may be taken (or used) indifferently for each other: or (which is the same thing) that a quantity which is increased or decreased only by an infinitely small quantity may be considered as remaining the same.

Responses to the difficulties: Maclaurin



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OF

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In Two BOOKS.

BY

COLIN MACLAURIN, A. M.

Professor of Mathematics in the University of Edinburgh, *and Fellow of the Royal Society.*

VOLUME I.

EDINBURGH:

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MDCCXLII.

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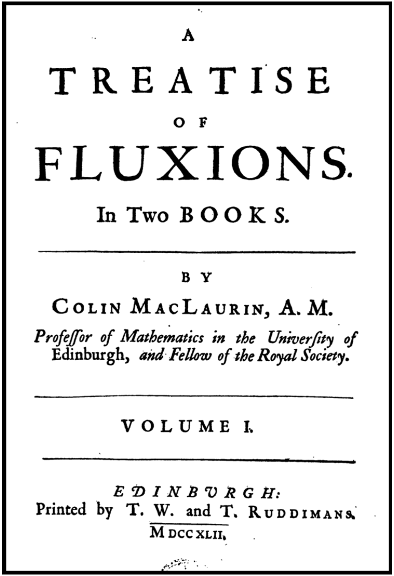
Responses to the difficulties: Maclaurin



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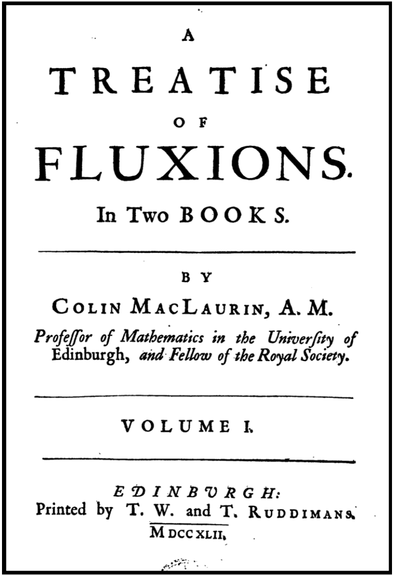

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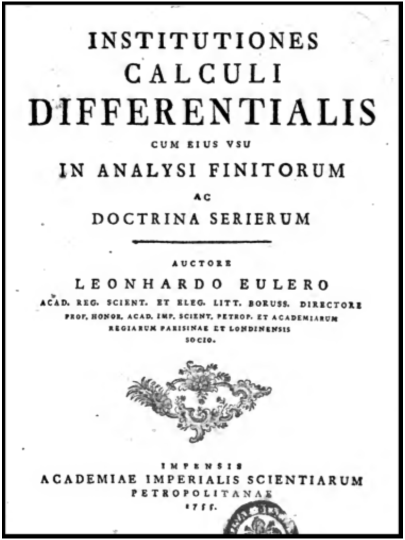
Colin Maclaurin (1742)

- written in direct response to Berkeley
- attempted to prove all propositions of calculus by classical Archimedean methods

  ('double contradiction': derive a contradiction from the assumption that $a > b$; derive a contradiction from the assumption that $b > a$; then it must be the case that $a = b$).

Responses to the difficulties: Euler


INSTITUTIONES
CALCULI
DIFFERENTIALIS
CUM EIUS VSU
IN ANALYSI FINITORUM
AC
DOCTRINA SERIERUM

AUCTORE
LEONHARDO EULERO
ACAD. REG. SCIENT. ET ELEG. LITT. BORUSS. DIRECTORE
PROF. HONOR. ACAD. IMP. SCIENT. PETROP. ET ACADEMIARUM
REGIARUM PARISINAE ET LONDINENSIS
SOCIO.

IMPENSIS
ACADEMIAE IMPERIALIS SCIENTIARUM
PETROPOLITANAE
1755.

Leonard Euler (1755):

> *An infinitely small quantity is nothing other than a vanishing quantity, and is therefore really equal to 0.*
>
> ...
>
> *If there occur different infinitely small quantities dx and dy, although both are equal to 0, nevertheless their ratio is not constant.*

Responses to the difficulties: Landen

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Concerning the

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A NEW BRANCH of the

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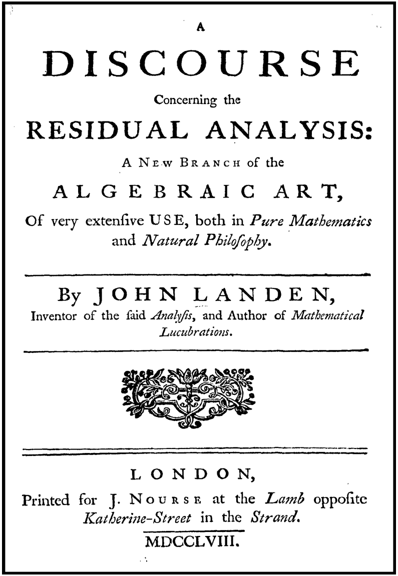
MDCCLVIII.

John Landen (1758)

Responses to the difficulties: Landen



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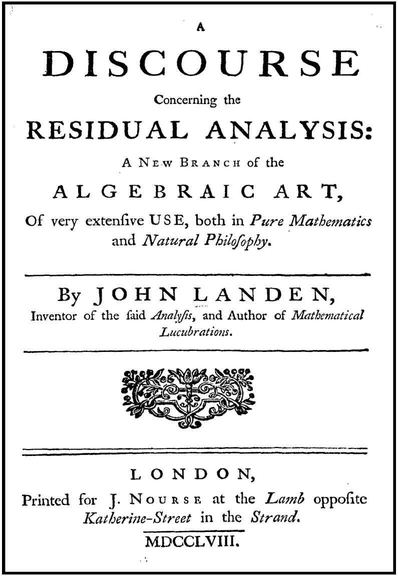
John Landen (1758)

- 'Fluxions are not immediately applicable to algebraic quantities ...'

Responses to the difficulties: Landen



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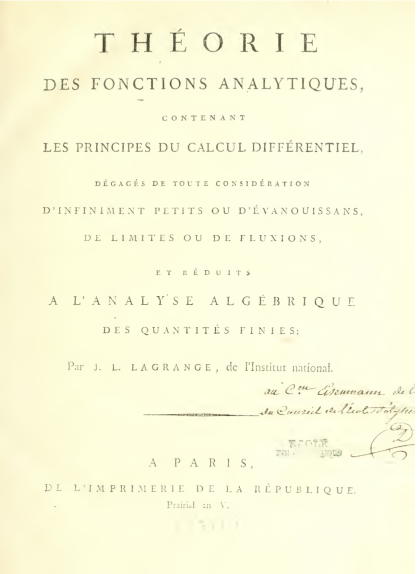
John Landen (1758)

- 'Fluxions are not immediately applicable to algebraic quantities ...'
- attempted a purely algebraic development of calculus

Responses to the difficulties: Lagrange



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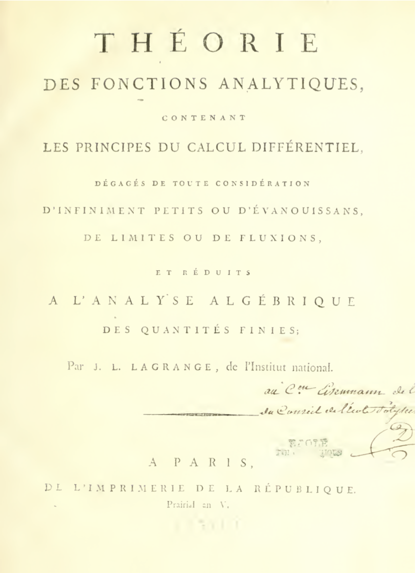
Responses to the difficulties: Lagrange



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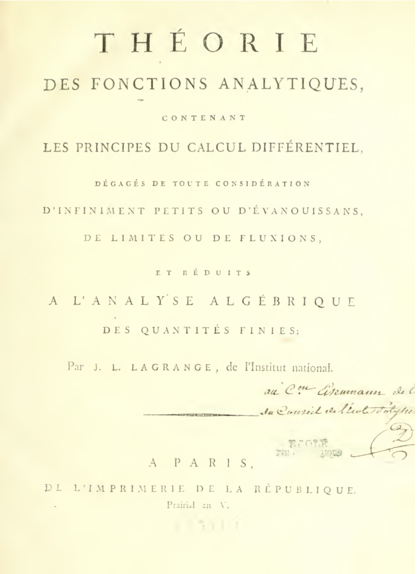
Another attempt to avoid 'infinitely small quantities'

Responses to the difficulties: Lagrange



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Joseph-Louis Lagrange (1797)

Another attempt to avoid 'infinitely small quantities'

(by taking functions to be defined by power-series expansions)

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Finally, note the increasing ‘professionalisation’ of mathematics in the eighteenth century:

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  - Lagrange followed Euler to Berlin, later went to Paris;

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  - Lagrange followed Euler to Berlin, later went to Paris;
- each Academy had its own ‘Mémoires’ or ‘Transactions’ enabling wider (and sometimes faster) circulation of new ideas.