

BO1 History of Mathematics  
Lecture VII  
Infinite series  
Part 1: A non-Western prelude

MT 2020 Week 4

# Summary

## Part 1

- ▶ A non-Western prelude

## Part 2

- ▶ Newton and the Binomial Theorem
- ▶ Other 17th century discoveries
- ▶ Ideas of convergence

## Part 3

- ▶ Much 18th century progress: power series
- ▶ Doubts — and more on convergence

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# Tantrasamgraha (1501)

Completed by Kelallur Nilakantha Somayaji (1444–1544) in 1501;  
concerns astronomical computations



## Keralan series

Infinite series for trigonometric functions appear in Sanskrit verse in an anonymous commentary on the *Tantrasamgraha*, entitled the *Tantrasamgraha-vyakhya*, of c. 1530:

दृष्टज्यात्रिज्ययोर्घातात् कोटचाप्तं प्रथमं फलम् ।  
ज्यावर्गं गुणकं कृत्वा कोटिवर्गं च हारकम् ॥  
प्रथमादिफलेभ्योऽथ नेया फलततिर्मुहुः ।  
एकत्रचाद्योज संख्याभिर्भक्ते ष्वेतेष्वनुक्रमात् ॥  
ओजानां संयुतेस्त्यक्त्वा युग्मयोगं धनुर्भवेत् ।  
दोःकोटघोरल्पमेवेह कल्पनीयमिह स्मृतम् ।  
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Proof supplied by Jyeṣṭhadeva in his *Yuktibhāṣā* (1530)

## Keralan series

From the *Tantrasamgraha-vyakhya*:

The product of the given Sine and the radius divided by the Cosine is the first result. From the first, [and then, second, third] etc., results obtain [successively] a sequence of results by taking repeatedly the square of the Sine as the multiplier and the square of the Cosine as the divisor. Divide [the above results] in order by the odd numbers one, three, etc. [to get the full sequence of terms]. From the sum of the odd terms, subtract the sum of the even terms. [The results] become the arc. In this connection, it is laid down that the [Sine of the] arc or [that of] its complement, which ever is smaller, should be taken here [as the given Sine]; otherwise, the terms obtained by the [above] repeated process will not tend to the vanishing magnitude.

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Modern interpretation:

$$R\theta = \frac{R(R \sin \theta)^1}{1(R \cos \theta)^1} - \frac{R(R \sin \theta)^3}{3(R \cos \theta)^3} + \frac{R(R \sin \theta)^5}{5(R \cos \theta)^5} - \dots$$

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But these results were unknown in the West until the 1830s

As we will see, the series for arctan was reproduced independently in Scotland in the 1670s

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XXXIII. *On the Hindú Quadrature of the Circle, and the infinite Series of the proportion of the circumference to the diameter exhibited in the four Sāstras, the Tantra Sangraham, Yucti Bhāshā, Carana Padhati, and Sadratnamāla. By CHARLES M. WILSE, Esq., of the Hon. East-India Company's Civil Service on the Madras Establishment.*

(Communicated by the MADRAS LITERARY SOCIETY and AUXILIARY ROYAL ASIATIC SOCIETY.)

Read the 13th of December 1832.

A'RYAB'HATTA, who flourished in the beginning of the thirty-seventh century of the *Cālī Yuga*,\* of which four thousand nine hundred and twenty years have passed, has in his work, the *AryaB'hatiyam*, in which he mentions the period of his birth, exhibited the proportion of the diameter to the circumference of the circle as 20000 to 62832, in the following verse :

*Chaturādikaṃ satamaṣṭagaṇanādvāśaśūtatā sāhasraṇāṃ  
Aṅgatawaya viśaṅbāhāsyasannō vṛitta pariṇāhāt*

Which is thus translated :

“ The product of one hundred increased by four and multiplied by eight, added to sixty and two thousands, is the circumference of a circle whose diameter is twice ten thousand.”

The author of the *Līlāvati*, who lived six centuries after A'RYAB'HATTA, states the proportion as 7 to 22, which, he adds, is sufficiently exact for common purposes. As a more correct or precise circumference, he proposes that the diameter be multiplied by 3927, and the product divided by 1250; the quotient will be a very precise circumference. This proportion is the same with that of A'RYAB'HATTA, which is less correct than that of

\* Or the sixth century of the Christian era.

† This verse is in the variety of the *Arjavṛittam* measure, called *Tīpālā*.