BO1 History of Mathematics Lecture VII Infinite series Part 2: The 17th century

MT 2020 Week 4

Infinite series 1600–1900: an overview

Lecture VII:

- mid–late 17th century: many discoveries
- early 18th century: much progress
- later 18th century: doubts and questions

▲□▶ ▲□▶ ▲□▶ ▲□▶ ■ ●の00

Infinite series 1600–1900: an overview

Lecture VII:

- mid–late 17th century: many discoveries
- early 18th century: much progress
- later 18th century: doubts and questions

Lecture VIII:

- early 19th century: Fourier series
- early 19th century: convergence better understood

▲□▶ ▲□▶ ▲□▶ ▲□▶ □ のQで

#### Newton and the general binomial theorem

#### CUL Add. MS 3958.3, f. 72

(See lecture IV)

If lab is an Hyperfile; eds, ch its symplotes, also. le + 10x2 + 10x3 + 5x4 + x5 %). The. times proceeding this prographion. \* =adel. T x+ 3×× +x3, x+2×× + 3×3 + ×+ x+4×× + & first avia is also inserted. The composition 1. Acducid from hence; vire: The same of igure above it is equall to y' . By wel table it may appeare of of y' Hyperbola adeb -x" +x7 -x8 + x9 - ×10 8te Suppor at adek abe a civele age a Parafola 81 = x . Mall fr=1= y lines fr. be progression in with 1 VI-XX. 1-XX. 1-XX/1-XX. 1-2XX+X4. 1-2XX+X4/1-XX 1-3xx + 3xt =-x6. Ve. Then will this avias fire, bars, gade, in this propristion. x. \*. x- \*\*\* \*. x- =x3+= xr. + \* x-4x3+6x5-4x7+x9 inserted. The proper above it 12. 6. after it save one. alles & per ion : are of this 22. 15. maconvall progressi formes 15. 60 ard. Tert. O. Tert. O. - Tert. O. Mar. 1. C. Ter Cond. and Asterset. intermidiate termes may be casily performed. The are ab 4th Collame 1. t. - + - + + (well propression may 1X1X-1X3X-2727 2X11413 X15 (c) Whereby is may appeare of what Since  $\Im_{L=X}$  is,  $\vartheta_{Y}^{L}$  avec abid is  $x - \frac{\pi}{24} - \frac{\pi}{24} - \frac{\pi}{112} \frac{J_{XY}}{H_{T}} - \frac{2\pi}{1112} - \frac{\pi}{11112} - \frac{\pi}{11112} \frac{\pi}{11112}$  etc. area aff is 22+25+25+12 (r.) Whereby also ye area & angle and may be found this. It arres of , all , agd , all ye are in this progression & N. 2+ \*\* \*\*\*\* - 2 - 2 + 5 × 7 + 5 × 7 + 5 + 8 - 18 × 5 \* 40×7 - 12 × 7 . 45 . cle in the following Track + 35-29 + 632 " . ye. And by this means having y' area abd, O. Tr. A. (with at age and gives) for since of at anyte Do may be from 0.- 11 0 Good: If Wax & Warmer = 16. y \* ekig an Hyputhia. +

# Recall: Newton's integration of $(1 + x)^{-1}$

	$(1 + x)^{-1}$	$(1 + x)^0$	$(1 + x)^1$	$(1 + x)^2$	$(1 + x)^3$	$(1 + x)^4$	
x	1	1	1	1	1	1	
$\frac{x^2}{2}$	-1	0	1	2	3	4	
$\frac{x^3}{3}$	1	0	0	1	3	6	
$\frac{x^4}{4}$	-1	0	0	0	1	4	
$\frac{x^5}{5}$	1	0	0	0	0	1	
:				- - -			·

The entry in the row labelled  $\frac{x^m}{m}$  and the column labelled  $(1 + x)^n$  is the coefficient of  $\frac{x^m}{m}$  in  $\int (1 + x)^n dx$ . (NB. Newton did not use the notation  $\int (1 + x)^n dx$ .)

▲□▶▲□▶▲□▶▲□▶ ▲□▶ ● ●

In fact, this method extends easily to any integer n

In fact, this method extends easily to any integer n

Newton's explanation:

The property of which table is  $y^t y^e$  sum of any figure and  $y^e$  figure above it is equal to  $y^e$  figure next after it save one. Also  $y^e$  numerall progressions are of these forms.

а	а	а	а	
b	a + b	2 <i>a</i> + <i>b</i>	3a + b	
с	b + c	a+2b+c	3a+3b+c	&с.
d	c + d	b+2c+d	a+3b+3c+d	
е	d + e	c + 2d + e	b+3c+3d+e	

- ロ ト - 4 回 ト - 4 □

(See: *Mathematics emerging*, §8.1.1.)

#### Newton and the general binomial theorem

#### CUL Add. MS 3958.3, f. 72

If lab is an Hyperfield ; eds. ch its strongholes , a also. le + Leve + 10×3 + rx4 + xr &c). The. times proceeding this propression. T x+ 3×× +x3, x+2×× + 3×3 + ×+ x+4×× + first area is also inserted. The composition 1. Acducid from hence; vire: The same of figuri follow igure above it is equall to y' By well table it may appeare of of y' Hyperbola adab -x + x7 - x8 + x? - x10 He Suppor at adek abe a civele age a Parafola . Arc Bez x. Stall fe=1= y lines fr. By 1-xx /1-xx. 1-+xx+x4. 1-+xx+x4/1-xx. 1 VI-XX. 1-XX. Then will their avias fire, Eads, gade, in this propristion. x. \*. x- \*\*\* \*. x- =x3+= xr. + Pade inte above it 12. 6. after it save one. alles & per . 15. necessival progressions are of this formes 15. 6Bard. 6. c. else, assige. Tert. O. Tert. O. - Tert. O. Mar. 1. C. Ter Cond. and Asterset. ana ap intermediate termes may be casily performed. The Collame 1. t. - t. I see (wel progression may XIX-IX3X-FX7X-9X11413X15 (fc) WREVERY if may appeare y, what x2X 4 X6 X8 X10X12X14X16518 (fc) - 7×11 - 11×13 - 11×15 avea abid is x - x3 - x5 - 27 - 5x9 So.) Whereby also ye area of angle and may be found this. It arres of , all , agd , all ye are in this progression & N. 2+ \*\* \* + 3 × 1 - 2 × 7 + 5 × 7 + × 4 × 1 × 3 - 18 × 5 + 40 × 7 - 12 × 2. 46 ch in the following Trach + 35-29 + 632 the offer by the mands having yt area abd, O. Tr. S. (with at agen and gives) for since of at anyte all may be from 0.- 11 0 Boyti Af B = x & Dyrexx = 16. y aking an Hyperbola. +

L. I. J. J. L. L. L. J. J. J. J. 一型X -1. -主· 0. 主· 1. 是· 2. 至· 3. 圣· 4. 是. 5. 些· 6. 1. =. 0. -= 0. =. 1. =. 3. 3. 3. 6. 5. 10. 9. 15. ☆X -1.- 音. 0. 音. 0. - 市. 0. 音. 1. 登. 4. 時. 10. - 10 11.4器. 0. 新. 0. 希. 0. 希. 0. 器. 1. 道: 5. 15. -1. 53 0. 105 0. -36. 0. 36. 0. -26. 0. 636. 1. 693. 6. 1. 231. 0. -945. 0. 2. 0. -5 0. 7 1024. 0. -24. 0. -1024. 0. - 1024. 0. -24. 0 For at a intermediate termes may

	$(1-x^2)^{-1}$	$(1-x^2)^{-\frac{1}{2}}$	$(1-x^2)^0$	$(1-x^2)^{\frac{1}{2}}$	$(1-x^2)^1$	$(1-x^2)^{\frac{3}{2}}$	$(1-x^2)^2$	
x	1	1	1	1	1	1	1	
$-\frac{x^3}{3}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	
x <sup>5</sup> 5	1	$\frac{3}{8}$	0	$-\frac{1}{8}$	0	3 	1	
$-\frac{x^7}{7}$	-1	$-\frac{5}{16}$	0	$\frac{3}{48}$	0	$-\frac{1}{16}$	0	
$\frac{x^9}{9}$	1	$\frac{35}{128}$	0	$-\frac{15}{384}$	0	$\frac{3}{128}$	0	
:		- - -			- - -	- - -	- - -	·

The entry in the row labelled  $\pm \frac{x^m}{m}$  and the column labelled  $(1 - x^2)^n$  is the coefficient of  $\pm \frac{x^m}{m}$  in  $\int (1 - x^2)^n dx$ .

(NB: possible slips in the last two rows of the original table)

Can fill in some initial values by other methods

Can fill in some initial values by other methods

Newton applied the formula

$$\binom{n}{k} = \frac{n(n-1)(n-2)\cdots(n-k+1)}{k!}$$

◆□▶ ◆□▶ ◆三▶ ◆三▶ 三回 のへぐ

to fractional n,

Can fill in some initial values by other methods

Newton applied the formula

$$\binom{n}{k} = \frac{n(n-1)(n-2)\cdots(n-k+1)}{k!}$$

to fractional n, so that

$$\binom{1/2}{1} = \frac{1}{2}, \quad \binom{1/2}{2} = \frac{1/2(1/2 - 1)}{2!} = -\frac{1}{8}$$

◆□▶ ◆□▶ ◆三▶ ◆三▶ 三回 のへぐ

and so on

#### Newton's and the general binomial theorem

Newton went on to extend this method to other fractional powers, and also to  $(a + bx)^n$ , thereby convincing himself of the truth of the general binomial theorem

▲□▶ ▲□▶ ▲□▶ ▲□▶ ■ ●の00

#### Newton's and the general binomial theorem

Newton went on to extend this method to other fractional powers, and also to  $(a + bx)^n$ , thereby convincing himself of the truth of the general binomial theorem — but this was not proved until the 19th century

▲□▶ ▲□▶ ▲□▶ ▲□▶ ■ ●の00

Newton went on to extend this method to other fractional powers, and also to  $(a + bx)^n$ , thereby convincing himself of the truth of the general binomial theorem — but this was not proved until the 19th century

On Newton and the binomial theorem, see https://www.youtube.com/watch?v=xv\_PWwdDWDk

#### One more table

#### The table at the bottom of the page gives the interpolations for $(1 + x)^n$ for half-integer *n*

If lab is an Hyperbola eds, ch its symplotes also. 10x2 + 10x3 + rx4 + xr &c). The. proceedition this prograpsion. \* =adel. 3×× +x1, x+2×× + 3×3 + ×4, x+4×× + first are is also inserter. The composition 1. Aducid from Rence; vir: The same igure above it is equall to y By well table it may appeare of y' Hyperbola adeb -x + x7 - x8 + x9 - ×10 8te Suppose of adek abe a civele age a Parafola 81 = x . Mall fr=1= y lines fr. By 1-xx /1-xx. 1-+xx+x4. 1-+xx+x4/1-xx. 1.VI-XX. 1-XX. Then will this areas fair, Gade, gade, preserigion . x. ¥ . x- \*\*\* +. x- =x3+= xr. + above it . 6. after it save one. alles of the nacoverale progressions are of this 15. formes 15. 60 ard. 0. 1024. 0. 111. 1. C. 242 C+22+2. 8+2c+31+2. Tert. ana ap intermediate termes may be casily performed. The 4th Collame 1. t. - t. I see (well proprission may x1x-1x3x-px7x-9x11413x15 (fc) WREVERy if may appears y, what x2x4 x6 x8 x10x12x14x16x16 sine & = x is, \$ y & area abid is x - \$ - x - - x - - x - 112 Se.) Whereby also ye area is angle and may see found this. It arres of , all, agd, all ye are in this progression & N. 30 2 1 + 2 + 23 - 18 × 5 + 40 × 7 - 12 × 9 . Ste che in the following Tach may see pereceived of all = {x+tx x + 1 x + 1 x x + 1 x x + 1 x x + 1 x x + 1 x x + 1 x x + 1 x x + 1 x x + 1 x x + 1 x x + 1 x x + 1 x x + 1 x x + 1 + 35-29 + 632 H. No. And by this means having y' area ald. to be we at ugh all gives the sine of at angle all may be friend 0. - 11 0 Boyel of B= x & Dyrexx = 16. y aking an Hyperbola. +

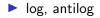
By further interpolations and integrations (based on strong geometric intuition) Newton found further series for:

▲□▶ ▲□▶ ▲ □▶ ▲ □▶ □ のへぐ

$$(1+x)^{p/q}$$

By further interpolations and integrations (based on strong geometric intuition) Newton found further series for:

(1 + x)<sup>$$p/q$$</sup>





By further interpolations and integrations (based on strong geometric intuition) Newton found further series for:

► 
$$(1+x)^{p/q}$$

log, antilog

sin, tan, ... (NB: cosine was not yet much in use)

▲□▶ ▲□▶ ▲□▶ ▲□▶ ■ ●の00

By further interpolations and integrations (based on strong geometric intuition) Newton found further series for:

► 
$$(1+x)^{p/q}$$

log, antilog

sin, tan, ... (NB: cosine was not yet much in use)

▲□▶ ▲□▶ ▲□▶ ▲□▶ ■ ●の00

▶ arcsin, arctan, ...

(See: Mathematics emerging, §§8.1.2–8.1.3.)

Newton on the move from finite to infinite series

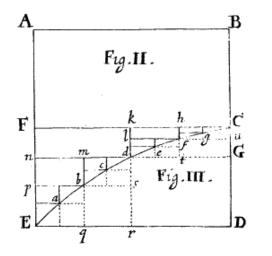
And whatever common analysis performs by equations made up of a finite number of terms (whenever it may be possible), this method may always perform by infinite equations: in consequence, I have never hesitated to bestow on it also the name of analysis.

(*De analysi*, 1669; Derek T. Whiteside, *The mathematical papers of Isaac Newton*, CUP, 1967–1981, vol. II, p. 241)

・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・

#### Other 17th-century discoveries (1a)

Brouncker, c. 1655, published 1668: area under the hyperbola given by  $\frac{1}{1 \times 2} + \frac{1}{3 \times 4} + \frac{1}{5 \times 6} + \cdots$ 



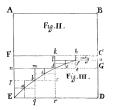
・ロットロット キョット キョン・ヨー わえの

Other 17th-century discoveries (1b)

$$E dCDE = \frac{1}{2x_3x_4} + \frac{1}{4x_5x_6} + \frac{1}{6x_7x_8} + \frac{1}{9x_{10}} & c.$$
  

$$E dCDE = \frac{1}{2x_3x_4} + \frac{1}{4x_5x_6} + \frac{1}{6x_7x_8} + \frac{1}{8x_9x_{10}} & c.$$
  

$$E dCyE = \frac{1}{2x_3x_4} + \frac{1}{4x_5x_6} + \frac{1}{6x_7x_8} + \frac{1}{8x_9x_{10}} & c.$$
  
(647)



And that therefore in the first feries balf the first term is greater than the fum of the two next, and half this fum of the fecond and third greater than the fum of the four next, and half the fum of those four greater than the fum of the next eight,  $\mathcal{C}_c$ , in infinitum. For  $\frac{1}{2} dD = br + bn$ ; but bn > fG, therefore  $\frac{1}{2} dD > br + fG$ ,  $\mathcal{C}_c$ . And in the fecond feries half the first term is lefs then the fum of the two next, and half this fum lefs then the fum of the four next,  $\mathcal{C}_c$  in infinitum.

That the first fories are the even terms, viz. the  $a^{4}$ ,  $b^{6}$ ,  $b^{6}$ ,  $b^{7}$ ,  $10^{6}$ ,  $c^{2}$ , and the fecond, the odd, viz. the  $1^{4}$ ,  $3^{4}$ ,  $5^{60}$ ,  $7^{60}$ ,  $9^{60}$ ,  $6^{60}$ ,  $8^{70}$ ,  $10^{60}$ ,  $c^{2}$ , and the fecond, the odd, viz. the  $1^{4}$ ,  $3^{4}$ ,  $5^{60}$ ,  $7^{60}$ ,  $9^{60}$ ,  $c^{60}$ ,  $8^{70}$ ,  $10^{60}$ ,  $c^{20}$ ,  $10^{10}$ ,  $c^{21}$ ,  $\frac{1}{100}$ ,  $\frac{1}{100$ 

ning, and  $\frac{1}{a-1}$  the fum of the reft to the end.

That  $\frac{1}{2}$  of the first terme in the *third* feries is lefs than the fum of the two next, and a quarter of this fum, lefs than the fum of the four next, and one fourth of this last fum lefs than the next eight, I thus demonstrate.

Let a the 3" or laft number of any term of the first Column, viz: of Divifors,

## Other 17th-century discoveries (2)

Mercator's series (1668), found by long division:

$$\frac{1}{1+a} = 1 - a + aa - a^3 + a^4 (\&c.)$$

Gives rise to series for log



・ロト ・ 国 ト ・ ヨ ト ・ ヨ ト

# Other 17th-century discoveries (3)



James Gregory (1671): ► general binomial expansion



## Other 17th-century discoveries (3)



James Gregory (1671):

- general binomial expansion
- series for tan, sec, and others, including

$$\theta = \tan \theta - \frac{1}{2} \tan^3 \theta + \frac{1}{5} \tan^5 \theta - \cdots$$

for 
$$-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$$

# Other 17th-century discoveries (3)



James Gregory (1671):

- general binomial expansion
- series for tan, sec, and others, including

$$heta = an heta - rac{1}{2} an^3 heta + rac{1}{5} an^5 heta - \cdots$$
  
For  $-rac{\pi}{4} \le heta \le rac{\pi}{4}$ 

◆□▶ ◆□▶ ◆□▶ ◆□▶ □ ○ ○ ○

Gregory to Collins, 23rd November 1670:

I suppose these series I send here enclosed, may have some affinity with those inventions you advertise me that Mr. Newton had discovered.

(On Gregory's work, see: *Mathematics emerging*, §8.1.4.)

#### Other 17th-century discoveries (4)

Gottfried Wilhelm Leibniz (1675):

The area of a circle with unit diameter is given by

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \&c.$$

#### Other 17th-century discoveries (4)

Gottfried Wilhelm Leibniz (1675):

The area of a circle with unit diameter is given by

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \&c.$$

The error in the sum is successively less than  $\frac{1}{3}$ ,  $\frac{1}{5}$ ,  $\frac{1}{7}$ , etc.

Therefore the series as a whole contains all approximations at once, or values greater than correct and less than correct: for according to how far it is understood to be continued, the error will be smaller than a given fraction, and therefore also less than any given quantity. Therefore the series as a whole expresses the exact value.

(See: *Mathematics emerging*, §8.3.)

John Wallis (1656), Arithmetica infinitorum:

$$\Box = \frac{4}{\pi} = \frac{3 \times 3 \times 5 \times 5 \times 7 \times 7 \times \cdots}{2 \times 4 \times 4 \times 6 \times 6 \times 8 \times \cdots}$$

John Wallis (1656), Arithmetica infinitorum:

$$\Box = \frac{4}{\pi} = \frac{3 \times 3 \times 5 \times 5 \times 7 \times 7 \times \cdots}{2 \times 4 \times 4 \times 6 \times 6 \times 8 \times \cdots}$$

▲□▶ ▲□▶ ▲ □▶ ▲ □▶ □ のへぐ

(Determined that

$$\Box > \sqrt{\frac{3}{2}},$$

John Wallis (1656), Arithmetica infinitorum:

$$\Box = \frac{4}{\pi} = \frac{3 \times 3 \times 5 \times 5 \times 7 \times 7 \times \cdots}{2 \times 4 \times 4 \times 6 \times 6 \times 8 \times \cdots}$$

▲□▶ ▲□▶ ▲ □▶ ▲ □▶ □ のへぐ

(Determined that

$$\Box>\sqrt{\frac{3}{2}},\quad \Box<\frac{3}{2}\sqrt{\frac{3}{4}},$$

John Wallis (1656), Arithmetica infinitorum:

$$\Box = \frac{4}{\pi} = \frac{3 \times 3 \times 5 \times 5 \times 7 \times 7 \times \cdots}{2 \times 4 \times 4 \times 6 \times 6 \times 8 \times \cdots}$$

(Determined that

$$\Box > \sqrt{\frac{3}{2}}, \quad \Box < \frac{3}{2}\sqrt{\frac{3}{4}}, \quad \Box > \left(\frac{3\times3}{2\times4}\right)\sqrt{\frac{5}{4}},$$

▲□▶ ▲□▶ ▲ 三▶ ▲ 三▶ 三 のへぐ

and so on)

John Wallis (1656), Arithmetica infinitorum:

$$\Box = \frac{4}{\pi} = \frac{3 \times 3 \times 5 \times 5 \times 7 \times 7 \times \cdots}{2 \times 4 \times 4 \times 6 \times 6 \times 8 \times \cdots}$$

(Determined that

$$\Box > \sqrt{\frac{3}{2}}, \quad \Box < \frac{3}{2}\sqrt{\frac{3}{4}}, \quad \Box > \left(\frac{3\times 3}{2\times 4}\right)\sqrt{\frac{5}{4}},$$

▲□▶ ▲□▶ ▲ 三▶ ▲ 三▶ 三 のへぐ

and so on)

Brouncker (1668): grouping of terms

John Wallis (1656), Arithmetica infinitorum:

$$\Box = \frac{4}{\pi} = \frac{3 \times 3 \times 5 \times 5 \times 7 \times 7 \times \cdots}{2 \times 4 \times 4 \times 6 \times 6 \times 8 \times \cdots}$$

(Determined that

$$\Box > \sqrt{\frac{3}{2}}, \quad \Box < \frac{3}{2}\sqrt{\frac{3}{4}}, \quad \Box > \left(\frac{3\times 3}{2\times 4}\right)\sqrt{\frac{5}{4}},$$

▲□▶ ▲□▶ ▲ 三▶ ▲ 三▶ 三 のへぐ

and so on)

Brouncker (1668): grouping of terms

```
Leibniz (1675): 'alternating' series
```

Power series (infinite polynomials):

enabled term-by-term integration for difficult quadratures;

▲□▶ ▲□▶ ▲ □▶ ▲ □▶ □ のへぐ

Power series (infinite polynomials):

enabled term-by-term integration for difficult quadratures;

helped establish sine, log, ... as 'functions' (transcendental);

▲□▶ ▲□▶ ▲ 三▶ ▲ 三▶ 三 のへぐ

Power series (infinite polynomials):

enabled term-by-term integration for difficult quadratures;

helped establish sine, log, ... as 'functions' (transcendental);

encouraged a move from geometric to algebraic descriptions;

・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・

Power series (infinite polynomials):

enabled term-by-term integration for difficult quadratures;

helped establish sine, log, ... as 'functions' (transcendental);

encouraged a move from geometric to algebraic descriptions;

for Newton (and others) inextricably linked with calculus.

Power series (infinite polynomials):

enabled term-by-term integration for difficult quadratures;

helped establish sine, log, ... as 'functions' (transcendental);

encouraged a move from geometric to algebraic descriptions;

for Newton (and others) inextricably linked with calculus.

Power series rank with calculus as a major advance of the 17th century

#### Calculus and series combined

# Newton's treatise of 1671, published 1736

# METHOD of FLUXIONS

AND

INFINITE SERIES;

WITH ITS

Application to the Geometry of CURVE-LINES.

By the INVENTOR Sir ISAAC NEWTON, Kr. Late Prefident of the Royal Society.

Tranflated from the AUTHOR'S LATIN ORIGINAL not yet made publick.

To which is fubjoin'd, A PERPETUAL COMMENT upon the whole Work,

Confiling of ANNOTATIONS, ILLUSTRATIONS, and SUPPLEMENTS,

In order to make this Treatine A compleat Inflitution for the use of LEARNERS.

By JOHN COLSON, M.A. and F.R.S. Mafter of Sir Joseph Williamsen's free Mathematical-School at Rockoffer.

LONDON: Printed by HENRY WOODFALL; And Sold by JOHN NOWSE, at the Lamb without Temple-Bar. M.DCC XXXVI. dam

23.12