BO1 History of Mathematics Lecture XIII Complex analysis Part 1: Complex numbers

MT 2020 Week 7

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Summary

Part 1

Complex numbers: validity and representation

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Substitution of complex values for real

Part 2

- Cauchy's contributions
- Riemann
- What is an analytic function?

Before 1600, very faint beginnings:

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Cardano (1545) [from quadratics]



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Bombelli (1572) [from cubics]

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- Cardano (1545) [from quadratics]
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- Harriot (c. 1600) [from quartics]

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Cardano (1545) [from quadratics]

Bombelli (1572) [from cubics]

Harriot (c. 1600) [from quartics]

But:

For the most part such roots were ignored: negative roots were described merely as 'false', but complex roots as 'impossible'. (Mathematics emerging, p. 459.)



Problem: find two numbers that add to 10 and multiply to 40,

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eendo, Sé, ize aggregati minuas ac addas dimitium diuidenth. Exemplifi, inhoc calar, diuide 1 on idua parese, producentes 4,00 adde 25 quadrati dimitih 1 o a 4 o, nia e 5,00 kmus remines e Sé adde eriam e, habebis parese Recondum fimilitudinem, ae 6 eys 8 km e 6 mer. Minumeri differunti in 10,000 initell'itanui 15 ofelta 1860. Schunding progeediur Arithmetia (lubiditas, cuius hoce extremum ut disa, adoo et lubidis, att initelle.

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aut 40 gdruplu ad 10, quare nos uolumus quadruplum totius A B, igitur fiat A D, quadratum a c.dimidii a B.& ex A D auferatur quadruplum A B,abfq; numero, Re igitur re fidui, fi aliquid maneret, addita & detracta ex a c.oftenderet partes, at quia tale refidu

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um eft minus, ideo imaginaberis 10 m: 15, id eft differentiæ A D, & quadrupli A B, quam adde & minue ex A c,& habebis quafitum, feilis cet s pine vias mi 40, & smine vi as mi 40, feu spine mits, & e mate m: 1 5, duc 5 p:R: m: 1 5 in 5 m: R: m: 15, dimifsis incruciationibus, fit 25 m:m: 15, quod elt p: 15, igitur hoc productum eft 40, natu ra tame A D, non eft eadem cu natura 40, nec A B, quia fuperficies eft remota à natura numeri, & lineze, proximius | 5 pire m: 15

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plir, inhoc calu, diuide 1 oin duas partes, producenes 49, auot 25 quadrati dimidi 10 a ed 49, fie c, als huias partes, producenes 49, auot 25 s, labelsis partes fecundum fimilitudinem, ge 67 ps 68 ue 67 mer. At hinumeri differunti in 10,000 midi 1 faciuta 10,261 ue 66, Khourdip progrediur Arithmetica fubrilitas, cuius hoc extremumut dist, adeo edi fubrile, uti fin intile.

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But regarded such ideas as absurd and useless

PRIMO. 169 Ho trouato un'altra forte di R.c.legate molto differen ri dall'altre, laqual nafce dal Capitolo di cubo eguale à tanti, e numero, quando il cubato del terzo delli tanti è maggiore del quadrato della meta del numero come in effo Capitolo fi dimostrarà, laqual forte di z. q. hànel fuo Algorifmo diuerfa operatione dall'altre, e diuerfo nome ; per che quando il cubato del terzo del li tànti è maggiore del quadrato della metà del numero; lo eccefio loro non fi può chiamare ne più, ne meno, però lo chiamarò più di meno, quando egli fi doue rà aggiongere, e quando fi douerà cauare, lo chiamerò men di meno, e questa operatione è necessarijfima più che l'altre R.c. L.per rifpetto delli Capitoli di potenze di poteze, accompagnati co li cubi, ò tanti, ô con tutti due infieme, che molto più fonoli cafi dell'agguagliare doue ne nafce quefta forte di R. che quelli doue nasce l'altra, la quale parerà à molti più tofto fofiftica, che reale, e tale opinione hò tenuto anch'io, fin' che hò trouato la fua dimoitratione in lince (come fi dimostrarà nella dimostratione del detto Capitolo in superficie piana) e prima trattarò del Moltiplicare, ponendo la regola del più & meno. Più uia più di meno, fa più di meno. Meno uia più di meno, fà meno di meno.

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Meno uia più di meno, fà meno di meno. Più uia meno di meno, fà meno di meno. Meno uia meno di meno, fà più di meno. Più di meno uia più di meno, fà più. Meno di meno uia più di meno, fà più. Meno di meno uia mend ia meno fa meno.

Si

"Another sort of cube root much different from the former ..."

A D > A P > A B > A B >

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Si

"Another sort of cube root much different from the former ..."

Systematic rules:

più di meno via più di meno, fà meno $(\sqrt{-1} \times \sqrt{-1} = -1)$ meno di meno via più di meno, fà più $(-\sqrt{-1} \times \sqrt{-1} = 1)$

PRIMO. 169 Ho trouato un'altra forte di R.c.legate molto differen ti dall'altre, laqual nafce dal Capitolo di cubo eguale à tanti e numero, quando il cubato del terzo delli tanti è maggiore del quadrato della meta del numero come in effo Capitolo fi dimostrarà, laqual forte di z. q. hànel fuo Algorifmo diuerfa operatione dall'altre, e diuerfo nome ; per che quando il cubato del terzo del li tànti è maggiore del quadrato della metà del numero: lo ecceffo loro non fi può chiamare ne più ne meno, però lo chiamarò più di meno, quando egli fi doue rà aggiongere, e quando fi douerà cauare, lo chiamerò men di meno, e questa operatione è necessariissima più che l'altre R.c. L.per rifpetto delli Capitoli di potenze di potéze, accompagnati có li cubi, ò tanti, ô con tutti due infieme, che molto più fonoli cafi dell'agguagliare doue ne nafce quefta forte di R. che quelli doue nasce l'altra, la quale parerà à molti più tofto fofiftica, che reale, e tale opinione hò tenuto anch'io, fin' che hò trouato la fua dimoitratione in lince (come fi dimostrarà nella dimostratione del detto Capitolo in superficie piana) e prima trattarò del Moltiplicare, ponendo la regola del più & meno.

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"Another sort of cube root much different from the former ..."

Systematic rules:

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But complex numbers were not admitted as solutions of equations — they could appear in calculations, provided they cancelled out by the end

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But complex numbers were not admitted as solutions of equations — they could appear in calculations, provided they cancelled out by the end

Complex numbers justified through practical use?

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Unpublished manuscripts contain systematic treatment of complex roots of equations

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Unpublished manuscripts contain systematic treatment of complex roots of equations — but these were removed by his editors

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Add MS 6783 f. 156

Unpublished manuscripts contain systematic treatment of complex roots of equations — but these were removed by his editors

Cf. Harriot's *Artis analyticae praxis* (1931), pp. 14–15;

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J. 12.2. De granupious dynapismo tanomicano - edga - bina + bana + bana - edga - bina + cana 5-a Je-au 9790: ledt = + lita + beau - bunn + cita - Ifun - comm + anna. 1-a 5-a Jigan = bedf-bdfa + dfaa -cofa + beau ingo: bedf I + tofa - bean + bana. an I - If. tra = test + bita + Stan - bane it-ma = test + cita - bean - can - 15 0000 sigo: bedf=====lofa + beau + bana + anan a = 1. It bids + bidse 1-a 1+a 1+aa I 0000 Engo: bidf I + blfa - beau - bana that I but - but - buna + buna 15-ma I but + with - buna - cana sign: bidt = +bdfa + dfan - bunn ~ # / / 4 ~ = - l be-we I beds - bean + waar I boos an I be an = df. myo: bulf I + beau - anan . Ston I bedy - Stan - and = 0000 an tot. sigo: bids = -bcan + anna. --- Vf

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Unpublished manuscripts contain systematic treatment of complex roots of equations — but these were removed by his editors

Cf. Harriot's *Artis analyticae praxis* (1931), pp. 14–15; see:

Muriel Seltman & Robert Goulding, Thomas Harriot's Artis analyticae praxis: an English translation with commentary, Springer, 2007

Descartes and 'imaginaries'

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choient $\frac{1}{2}, 1, 4, \frac{1}{2}$, & que celles de la premiere efficient $\frac{1}{2}/3, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}/3$. Come orseal de cooperation peut aufly feruir pour rendre la quantité connait de quelqu'un des termes de l'Equatio égale come aquelque autre donnée, comme fiayant durant de come fiayant durant de come fiayant

LA GEOMETRIE.

mad vec Equation (D) yeut auoir en fa place vne autre Equation, en laquelfaite (D) yeut auoir en fa place vne autre Equation, en laquelfaite (D) yeut (D) yeu

Que les marrier pas toufioirs preclus, mais quelquelois feulement imaginer marrier pas toufioirs reclus, mais quelquelois feulement imaginer marrier quelquelois feulement imaginer quelquefois autone quantité, qui corresponde a celles quelquefois autone quantité, qui corresponde a celles marrier de la magine. Comme êncore qui on en puille imagine. comme êncore qui on en puille imaginer en routefois qu'in ercelle, qui et a. & pour les deux autone fois qu'in celle, qui et a. & pour les deux autone fois qu'in celle, qui et a. & pour les deux autone fois qu'in celle, qui et a. & pour les deux autore, quoy qu'on les augmente, ou diminue, ou moltplie en la façon que je viens d'expliquer, on ne fçauroie les rendreatures qu'in maginaires.

Livésprobleme, ovient avne Equation, en l'aquelle la quanprobleme, ovient avne Equation, en l'aquelle la quanté inconne a trois dimensions, en l'aquelle la quanprobleme, autoris dimensions, en l'aquelle la quanprobleme, autoris dimensions, en la quelle la quanté inconne a trois dimensions, en la quelle la quanne dimensione, en la quelle la quanne dimensione, en la quelle la quelle la quanne dimensione, en la quelle la quelle la quelle la quelle nombres trois dimensione, en la quelle la quelle la quelle nombres trois dimensione, en la quelle la quelle la quelle nombres trois dimensione, en la quelle la quelle nombres trois dimensione, en la quelle la quelle nombres trois dimensione, en la quelle la quelle la quelle molteners dimensione, en la quelle la quelle la quelle molteners dimensione, en la quelle la quelle la quelle molteners dimensione, en la quelle la quelle la quelle molteners dimensione, en la quelle la quelle la quelle molteners dimensione, en la quelle la quelle la quelle la quelle molteners dimensione, en la quelle la quelle la quelle la quelle molteners dimensione, en la quelle la quell

La géométrie (1637):

introduced the term 'imaginaire'

Descartes and 'imaginaries'

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choient $\frac{1}{2}, 1, 4, \frac{1}{2}$, & que celles de la premiere efficient $\frac{1}{2}/3, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}/3$. Came Game Game Came aguelle tité connué de quelqu'un des termes de l'Equatio égale came aquelque autre donnée, comme fiayant de rest $\frac{1}{2}, \frac{1}{2}, \frac{1}{2$

LA GEOMETRIE.

E la quantité connuë, du terme qui occupe la troifiefme elle autre place, a fçauoir celle qui eft icy *b b*, foit 3 *a a*, il faut fupporent. fer y $\infty x \sqrt{\frac{1}{2a}}$, puis effcrire $y^{+a} - \frac{3}{2}ady + \frac{1}{2a} \sqrt{\frac{1}{2}} x^{2}$, $x \infty e$.

Que les marrier pas touffour seelles, mais quelquelois feulement imaginations autont quelquelois feulement imaginer autont quelquelois quelquelois feulement imaginer quelquelois autoin chafque Equation, mais qu'il n'y a marrier quelquelois accume quanties, qu'o car puiffe imaginer roiseit celle cy', s' $-4s^2 + 13 x = 1030$, il n'y en arounefois qu'in crelle, qu'il cit, s, pour les deux autres, quoy qu'onles augmente, ou diminae, ou multaple en la façon que je viens d'expliquer, on ne feavoir les reoffea autres qu'in maginaires.

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La géométrie (1637):

introduced the term 'imaginaire' — meant to be derogatory?

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Descartes and 'imaginaries'

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LA GEOMETRIE. eftoient 1, 1, & 1, & cque celles de la premiere eftoient +V3, +V3, & +V3. Cóment Cete operation peut auffy feruir pour rendre la quantité connuë de quelqu'un des termes de l'Equatio efgale a quelque autre donnée, comme fi avant -- bbx+c1200 On veut auoir en fa place vne autre Equation, eu laquel-

alea le la quantité connuë, du terme qui occupe la troifiefme autre place, a fçauoir celle qui eft icy bb, foit 3 a a, il faut fuppofery 20 x V 344 puisefcrire y 1 - 3 ady + 1 1 3 200.

Que les Au refte tant les vrayes racines que les fauffes ne font pas toufiours reelles; mais quelquefois feulement imaginaires, c'eft a dire qu'on peut bien touliours en imaginer couent autant que iay dit en chalque Equation; mais qu'il n'y a fire recl- quelquefois aucune quantite, qui corresponde a celles maginai- qu'on imagine. comme encore qu'on en puiffe imaginertroisch celle cy, x' - 6xx+ 13x- 1020, il n'y en a toutefois qu'vne reelle, qui est 2, & pour les deux autres, quoy qu'on les augmente, ou diminue, ou multiplie en la façon que ie viens d'expliquer, on ne scauroie les rendre autres qu'imaginaires.

Or quand pour trouver la construction de quelque Laredaproblefme, on vient a vne Equation, en laquelle la quan-tité inconnuë a trois dimensions ; premierement si les quantites connues , quiyfont , contienent quelques nombres rompus, il les faut reduire a d'autres entiers, par la multiplication tantost expliquée ; Et s'ils en contienent de fours , il faut apffy les reduire a d'autres rationaux, autant qu'il fera poffible, tant par cete meime muttiplication,

La géométrie (1637):

introduced the term 'imaginaire' — meant to be derogatory?

Didn't regard them as numbers

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Ideas about complex numbers in the later 17th century

John Wallis, *A treatise of algebra* (1685): complex numbers based on insights derived from

Euclidean geometry

trigonometry

properties of conics

(See: *Mathematics emerging*, §15.1.1.)



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A man starts at A and walks 5 yds to B, then retreats 2 yds to C: overall, he has covered 3 yds.

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A man starts at A and walks 5 yds to B, then retreats 2 yds to C: overall, he has covered 3 yds. If he instead retreats 8 yds to D, then we may say that he has covered -3 yds.

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- Somewhere on the seashore, we gain 26 units of land from the sea, but lose 10 units.



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- Somewhere on the seashore, we gain 26 units of land from the sea, but lose 10 units. Thus, we have gained 16 units overall;



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- Somewhere on the seashore, we gain 26 units of land from the sea, but lose 10 units. Thus, we have gained 16 units overall; if this is a perfect square, then it has side 4 units of length.



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- Somewhere on the seashore, we gain 26 units of land from the sea, but lose 10 units. Thus, we have gained 16 units overall; if this is a perfect square, then it has side 4 units of length.
- If instead we lose 26 units of land, but gain 10, then we have lost 16 units overall, or gained -16.



- A man starts at A and walks 5 yds to B, then retreats 2 yds to C: overall, he has covered 3 yds. If he instead retreats 8 yds to D, then we may say that he has covered -3 yds.
- Somewhere on the seashore, we gain 26 units of land from the sea, but lose 10 units. Thus, we have gained 16 units overall; if this is a perfect square, then it has side 4 units of length.
- ► If instead we lose 26 units of land, but gain 10, then we have lost 16 units overall, or gained -16. The area in question (assumed to be a square) might therefore be viewed as having side √-16.

(see: Leo Corry, A brief history of numbers, OUP, 2015, pp. 184–185)

Wallis: imaginary numbers as geometric means

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Wallis: imaginary numbers as geometric means



(see: Leo Corry, *A brief history of numbers*, OUP, 2015, pp. 185–186)

"A new Impossibility in Algebra"

John Wallis, *A treatise of algebra*, p. 267 'Of negative squares': ... requires a new Impossibility in Algebra



Which gives indeed (as, before) a double value of AB, $\sqrt{175}$, $-\sqrt{-81}$, and $\sqrt{175}$, $-\sqrt{-81}$: But fuch as requires a new Impoffibility in Algebra, (which in Lateral Equations doth not happen;) not that of a Negative Root, or a Quantity lefs than nothing; (as before,) but the Root of a Negative Square. Which in ftrictnefs of fpeech, cannot be: fince that no Real Root (Affirmative or Negative,) being Multiplied into itfelf, will make a Negative Square.

Complex numbers in the 18th century (1)



Nature remained unclear:

"that amphibian between being and not-being, which we call the imaginary root of negative unity" (Leibniz, 1702)

But complex numbers were increasingly being used ...

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Complex numbers in the 18th century (2)

206 MEMOIRES DE L'ACADEMIE ROYALE boles, dépend en partie de la quadrature du cercle, & en partie de la quadrature de l'hyperbole ou de la defcription de la Logarithmique. Maniéres abrégées de transformer les différentielles composées en simples , & réciproquement ; Et même les simples imaginaires en réelles composées. PROBL. I. Transformer la différentielle dat en une différentielle Logarithmique a dr., & réciproquement. Faires $z = \frac{t-1}{t+1} \times b$, & yous aurez $\frac{a d z}{b b-z z} = \frac{a d t}{z b t}$. Réciproquement prenez $t = \frac{+z+b}{-z+b}$, & yous aurez $\frac{adz}{zbz} =$ = adz Corol. On transformera de même la différentielle adz en $\frac{-adt}{2bt\sqrt{-1}}$ différentielle de Logarithme imaginaire; & réciproquement. **PROBL.** II. Transformer la différentielle $\frac{a d z}{b b + z z}$ en différentielle de fecteur ou d'arc circulaire $\frac{-aat}{v_1 - bbrt}$; & réciproquement. Faites $z = V \frac{1}{1} - bb$, & vous aurez $\frac{a dz}{bb + zz} = \frac{-a dt}{\sqrt{1 - bb dz}}$ Réciproquement prenez $t = \frac{1}{r_1 + h^2}$, & vous aurez $\frac{-a\,dz}{zVz-bbcz} = \frac{a\,dz}{b\,b+zz}$ PROBL. III. Transformer la différentielle ddx en différentielle de secteur hyperbolique ad ; & réciproquement. Faires $z = \sqrt{\frac{1}{t} + bb}$, & enfuire $t = \frac{1}{bb - xt}$; & yous aurez ce qu'on demande. PROBL

Johann Bernoulli, 'Solution d'un problème concernant le calcul intégrale, ...', *Mémoires de l'Académie royale des sciences*, 1702:

Complex numbers in the 18th century (2)

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Maniéres abrégées de transformer les différentielles composées en fimples, & réciproquement; Et même les simples imaginaires en réelles composées.

PROBL. I. Transformer la différentielle $\frac{ddz}{bb-zz}$ en une différentielle Logarithmique $\frac{ddz}{bb}$, & réciproquement. Faites $z = \frac{t-1}{t+1}xb_0$ & vous aurez $\frac{ddz}{bb-zz} = \frac{ddz}{bb}$. Réciproquement prenez $t = \frac{+z+b}{-z+b}$, & vous aurez $\frac{ddz}{bb-zz} = \frac{ddz}{zbt} =$ $= \frac{ddz}{bb-zz}$.

Corol. On transformera de même la différentielle $\frac{ddz}{b+z}$ z en $\frac{-zdz}{tbt/-1}$ différentielle de Logarithme imaginaire; & réciproquement.

PROBL II. Transformer la différentielle $\frac{dt}{dt+z_n}$ en différentielle de fecteur ou d'arc circulaire $\frac{-s dt}{z\sqrt{t-bt}}$; éc réciproquement.

Faites $z = \sqrt{\frac{1}{1-bb}}$, & vous aurez $\frac{adz}{bb+xz} = \frac{-adt}{y\sqrt{-bbt}}$ Réciproquement prenez $t = \frac{1}{zz+bb}$, & vous aurez $\frac{-adz}{y\sqrt{-bbt}} = \frac{adz}{b+xz}$.

PROBL III. Transformer la différentielle $\frac{ads}{bb-ss}$ en différentielle de fecteur hyperbolique $\frac{adt}{avi+bbit}$; & réciproquement.

Faites $z = \sqrt{\frac{1}{t} + bb}$, & enfuite $t = \frac{1}{bb - zz}$; & vous aurez ce qu'on demande. PROBL Johann Bernoulli, 'Solution d'un problème concernant le calcul intégrale, ...', *Mémoires de l'Académie royale des sciences*, 1702:

by making the substitution $z = \sqrt{\frac{1}{t} - bb}$, transform the differential $\frac{adz}{bb+zz}$ into $\frac{-adt}{2bt\sqrt{-1}}$

Complex numbers in the 18th century (2)

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PROBL. III. Transformer la différentielle $\frac{ads}{bb-sc}$ en différentielle de fecteur hyperbolique $\frac{ads}{svi+bbin}$; & réciproquement.

Faires $z = \sqrt{\frac{1}{i} + bb}$, & enfuire $t = \frac{1}{bb - zz}$; & yous aurez ce qu'on demande. PROBL Johann Bernoulli, 'Solution d'un problème concernant le calcul intégrale, ...', *Mémoires de l'Académie royale des sciences*, 1702:

by making the substitution $z = \sqrt{\frac{1}{t} - bb}$, transform the differential $\frac{adz}{bb+zz}$ into $\frac{-adt}{2bt\sqrt{-1}}$

No worries about the validity of switching between real and complex integrals

(See *Mathematics emerging*, §15.2.1)

Complex numbers in the 18th century (3)

[192]

How ÆQUATIONS are to be folu'd.

A FTER therefore in the Solution of a Queflion you are come to an Acquation, and that Acquation is duly reduc'd and order'd ; when the Quantitics which are fuppos'd given, are really given in Numbers, those Numbers are to Se fubfitured in their room in the Equation, and you'll have a Numeral Equation, whole Root being extracted will fatisfy the Queflion. As if in the Division of an Angle into five equal Parts, by putting r for the Radius of the Circle, a for the Chord of the Complement of the propos'd Angle to two right ones, and z for the Chord of the Complement of the fifth Part of that Angle, I had come to this Equation, x'-srrx'+sr*x-r'q=0. Where in any particular Cafe the Radius r is given in Numbers, and the Line q fubtending the Complement of the given Angle; as if Radius were 10; and the Chord 2; I fubfitute those Numbers in the Equation for r and q, and there comes out the Numeral Acquation x' - 500x' + 50000 - 30000 = o, whereof the Root being extracted will be x, or the Line fubtending the Complement of the fifth Part of that given Angle.

But the Root is a Nomber which being fubfitured in the Equation to the Letter or Species fignifying of the Nature the Root, will make all the lettern vanifufor he Nature the Thus Unity is the Root of the Equation x^{-1} as Equation: $-e^{-1} - igx + 4gx - 30 = 0$, becaufe being write for xit produces 1 - 1 - 1 + 4g

 -2_0 , that is, notions. And thus, if for x you write the Number 3, or the Negative Number. $-x_0$, and in both Cafes there will be produced nothing, the Affinantive and Negative Terms in thefe four Cafes defronging one another 3; then fince any of the Numbers written in the Equation folish the Convolution of x, by making all the Terms of the Equation together equal to nothing, any of them will be the Root of the Equation.

And that you may not wonder that the fame Equation may have feveral Roots, you mult know that there may be more Solutions [than one] of the fame Problem. As if there was fought the Interfections of two given Circle; there are two Interfections and confequently the Quefilm admits two Anfwers; and then the Equation determining Isaac Newton, *Universal Arithmetick*, 1728:

p. 195: "it is just that the Roots of Equations should be often impossible, lest they should exhibit the cases of Problems that are impossible as if they are possible"

Complex numbers in the 18th century (3)

[192]

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p. 195: "it is just that the Roots of Equations should be often impossible, lest they should exhibit the cases of Problems that are impossible as if they are possible" — complex numbers as an indicator of real-world solvability of problems

Complex numbers in the 18th century (4)

Leonhard Euler also used them freely: e.g., in *Introductio in analysin infinitorum*, 1748, §138:

$$e^{+\nu\sqrt{-1}} = \cos .\nu + \sqrt{-1}.\sin .\nu$$

$$e^{-v\sqrt{-1}} = \cos v - \sqrt{-1} \cdot \sin v$$

(See Mathematics emerging, §9.2.3)

	DE OUANTATIBUS TRANSCENDENT.
	104 DE QUARTERE
L1B. I.	$(1+v\sqrt{-1})+(1-v\sqrt{-1})$
	; atque fin. v ==
	$(1 + \frac{v\sqrt{-1}}{v}) - (1 - \frac{v\sqrt{-1}}{v})$
	. In Capite autem
	præcedente vidimus effe (1 + -) =e, denotante e bafin
	Logarithmorum hyperbolicorum : fcripto ergo pro a partim
	+ v V - 1 partim - v V - 1 erit cof. v =
	+
	2 & fin.v =
	Ex quibus intelligitur quomodo quantitates exponentiales ima-
	ginariæ ad Sinus & Cofinus Arcuum realium reducantur. Erit
	vero e+vV-1 = cof. v+V-1. fm. v & e - vV-1 =
	cof. v - V - I. fin. v.
	139. Sit jam in iildem formulis §. 130. # numerus infinite
	parvus, feu n = 1, existente i numero infinite magno, erit
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	onim evanefcentis 2 Sinus eft infi zoualis, Cofinus vero
	Li - of the balance
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	2 (cof. 2+V - 1. fm. 2) - (cof. 2 - V - 1. fm. 2) Sn-
	1 2V-1
	mendis autem Logarithmis hyperbolicis fupra (115) oftendi-
	= 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1
	$x_{1115} = (1 + x)^{-1} (1 + x)^{-1} = 1, x_{11} = 1 + \frac{1}{1}$
	pointo

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Every polynomial equation of degree n has exactly n roots.

Every polynomial equation of degree n has exactly n roots.

Early 17th century: known that an equation of degree n may have n roots

Every polynomial equation of degree n has exactly n roots.

- Early 17th century: known that an equation of degree n may have n roots
- During 17th century: complex numbers gradually admitted as roots

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Every polynomial equation of degree n has exactly n roots.

- Early 17th century: known that an equation of degree n may have n roots
- During 17th century: complex numbers gradually admitted as roots
- 15 Sept 1759: Euler asserted theorem in a letter to Nicholas Bernoulli, but didn't prove it

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- 1806: new proof by Argand
- ▶ 1821: Argand's proof appears in Cauchy's *Cours d'analyse*



Darwerende gorfog augaare bet Sporgsmaat, foorban Directionen aualgetiff ber beregues, eller hverban rette Linier burde ubtreftes, naar af een euefte Bigning miltem een ubefeindte gegen ginve finier finibe fanne finibe er Ubtreft, ber forefulltebe baab ben wieftenberte Beungbe og bens Direction.

för nagatalese at finne before bette Einstginnad, lenger isg til Gumbwebt be Seminger, etc finne än ungentigta. Zur förler er at öre SDirece tienned Beanbring, ber ved elgetsaltfe Dereatiener fan formbringet, ogfan fand for Völtgeba, uten for fassivt ben ved algetsaltfe Dereatiener fan form Anabe en Utigeba, uten for fassivt ben ved algetsaltfe Dereatiener fan forandres. Min ab den ved bligt i fan forsandres i for in minde fastierte fan faswanligt Beettaring), uten til ben mohinter, eller fan soften bei faswanligt Beettaring), uten til ben mohinter, eller fan soften bei her hander. Maabe, og i Arnögt til be errige Direktmet sære unpleftigt. Dette er of osaa Caspar Wessel, 'Om Directionens analytiske Betegning ...' ['On the analytic representation of direction ...'], Nye Samling af det Kongelige Danske Videnskabers Selskabs Skrifter, 1799



Darwarende Borfog augaare bet Sporgsmaat, foordan Directionen analytiff ber betegnes, eller hoverban rette Linite burde ubtraftes, naar af ein eurfte Ligning miltm een ubefeindte gin geine finite filite funne finite et Ubtrof, ber forfelliebe baab ben ubefeindtes Bungbe og bens Direction.

Bor nagenlebes at finne before bette Gravigsmal, lenger is jit ill mub web to Seminer, etc finne im gungettigt. Den feite er at obte Direce tiennet Grandbring, ber ved elgebraitfe Derentiener fan frembringet, gafa her vod bres Segu at fereflitte. Den ondere at Direction en einem Okras fand her Villgebra, uben fer lasvitt ben ved algebraitfe Derentiener fan far andere. Mins da ben ved bijfe i ein forandres i for an inder eine farten wanlige Gettaring), uben til ben mobjate, eller fan seine ben schradere wanlag Gettaring), uben til ben mobjate, eller fan seine ben schradere Waabe, g i genigt til be ertige Pieblemet stere ueplefeligt. Dette er oste Caspar Wessel, 'Om Directionens analytiske Betegning ...' ['On the analytic representation of direction ...'], Nye Samling af det Kongelige Danske Videnskabers Selskabs Skrifter, 1799

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French translation published in 1897

ESSAI

SUR UNE MANIÈRE DE REPRÉSENTER

LES QUANTITÉS IMAGINAIRES

DASS

LES CONSTRUCTIONS GÉOMÉTRIQUES,

PAR R. ARGAND.

* ÉDITION PRÉCÉDÉE D'UNE PRÉVACE PAR M. J. HOÜEL Er serve s'es apressica Gostesant des Extrails de *Annales de Gergones*, relatifs à la question des inscinaires.

PARIS,

GAUTHIER-VILLARS, IMPRIMEUR-LIBRAIRE be bereat die Logaitedes, de l'école polytéchnique, successer de Mallet Acheflier, Qui de Argenting, 15.

1874 (Tous droits réservés.) Robert Argand, *Essay on a* method of representing imaginary quantities ..., 1806



Theory of Conjugate Functions, or Algebraic Couples ; with a Preliminary and Elementary Essay on Algebra as the Science of Pure Time.

By WILLIAM ROWAN HAMILTON,

M.R.I.A., F.R.A.S., Hon.M.R.S.Ed. and Dub., Fellow of the American Academy of Arts and Sciences, and of the Royal Northern Antiquarian Society at Copenhagen, Andrews' Professor of Astronomy in the University of Dublia, and Royal Astronome of Ireland.

Read November 4th, 1833, and June 1st, 1835.

General Introductory Remarks.

THE Study of Algebra may be pursued in three very different schools, the Practical, the Philelogical, or the Theoretical, according as Algebra itself is accounted an Instrument, or a Language, or a Contemplation ; according as ease of operation, or symmetry of expression, or clearness of thought, (the agers, the fari, or the supere, J is eminently prized and sought for. The Practical person seeks a Rule which he may apply, the Philological person seeks a Formula which he may write, the Theoretical person seeks a Theorem on which he may meditate. The felt imperfections of Algebra are of three answering kinds. The Practical Algebraist complains of imperfection when he finds his Instrument limited in power ; when a rule, which he could happily apply to many cases, can be hardly or not at all applied by him to some new case; when it fails to enable him to do or to discover something else, in some other Art, or in some other Science, to which Algebra with him was but subordinate, and for the sake of which and not for its own sake, he studied Algebra. The Philological Algebraist complains of imperfection, when his Language presents him with an Anomaly; when he finds an Exception disturb the simplicity of his Notation, or the symmetrical structure of his Syntax : when a Formula must be written with precaution, and a Symbolism is not universal. The Theoretical Algebraist complains of imperfection, when the clearness of his Contemplation is obscured; when the Reasonings of his Science seem anywhere to oppose each other, or become in any part too complex or too little valid for his belief to rest firmly upon them : or when, though trial may have taught him that a rule is useful, or that a formula gives true results, he cannot prove that rule, nor understand that formula : when he cannot rise to intuition from induction, or cannot look beyond the signs to the things signified.

Transactions of the Royal Irish Academy, 1837

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