Chapter 1

Introduction

We will show that many phenomena in ecology, biology, biochemistry and medicine can be modelling mathematically. We will learn how to formulate mathematical models of biological processes, how to analyse them using a range of techniques from applied mathematics, and then how to relate the outcomes of our mathematical analysis to the real-world system we are studying. We will focus initially on systems where spatial variation is either absent or, at least, not important. In such cases only the temporal evolution needs to be described, and typically this is done using ordinary differential equations that describe how the concentrations or densities evolve over time. We will study how to build and analyse these models in order to extract useful information from them. We will then consider models where there is explicit spatial variation, and so we need to build models that incorporate spatial effects. In ecological and biological applications the main physical phenomenon governing spatial variation is typically, but not exclusively, diffusion. Thus many models use systems of parabolic partial differential equations to describe biological mechanisms, again in terms of the evolution of concentration or density. We will study how to build and analyse models in this context. Finally, we will explore how to connect these macroscale models to more detailed models that take into account the behaviour of individuals within a system. In particular we will draw examples from:

- enzyme-substrate dynamics and other biochemical reactions;
- infectious disease epidemics;
- biological pattern formation;
- cell cycle dynamics and tissue growth;
- tumour growth.

The main references for this lecture course will be:

- J. D. Murray, *Mathematical Biology, Volume I: An Introduction*, 3rd Edition (Springer, 2002);
- J. D. Murray, *Mathematical Biology, Volume II: Spatial Models and Biomedical Applications*, 3rd edition (Springer, 2003);
- L. Edelstein-Keshet, *Mathematical Models in Biology* (SIAM, 2005).

Other useful references include (but are no means compulsory):

- J. Keener and J. Sneyd, *Mathematical Physiology*, First Edition (Springer, 1998);
- N. F. Britton, *Essential Mathematical Biology* (Springer, 2003).

Additional references that might provide some useful reading are provided with the text.