Tuesday, 10 November 2020 11:27

Viscous Flow Lecture 10

last time: boundary layer separation at x_s where the shear stress vanishes and the flow reverses.

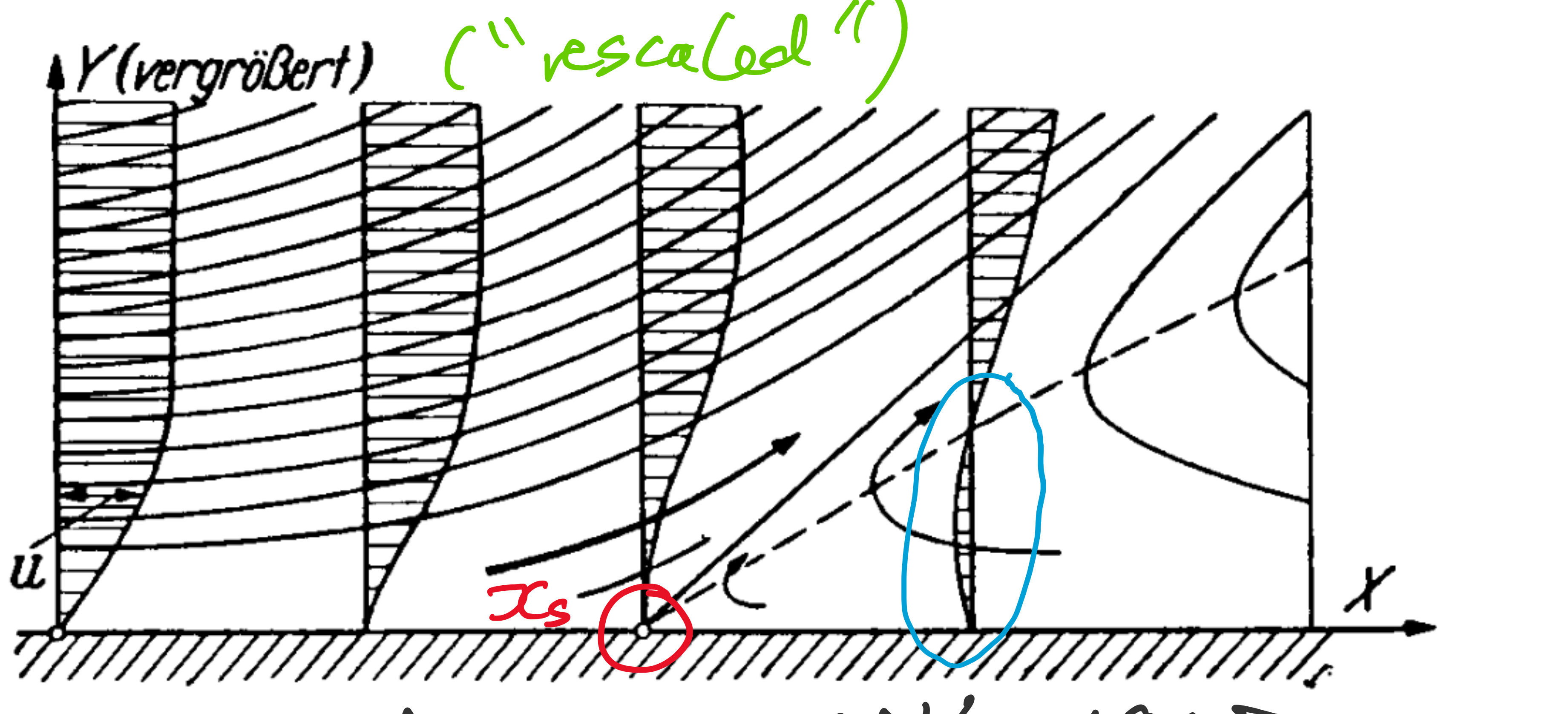
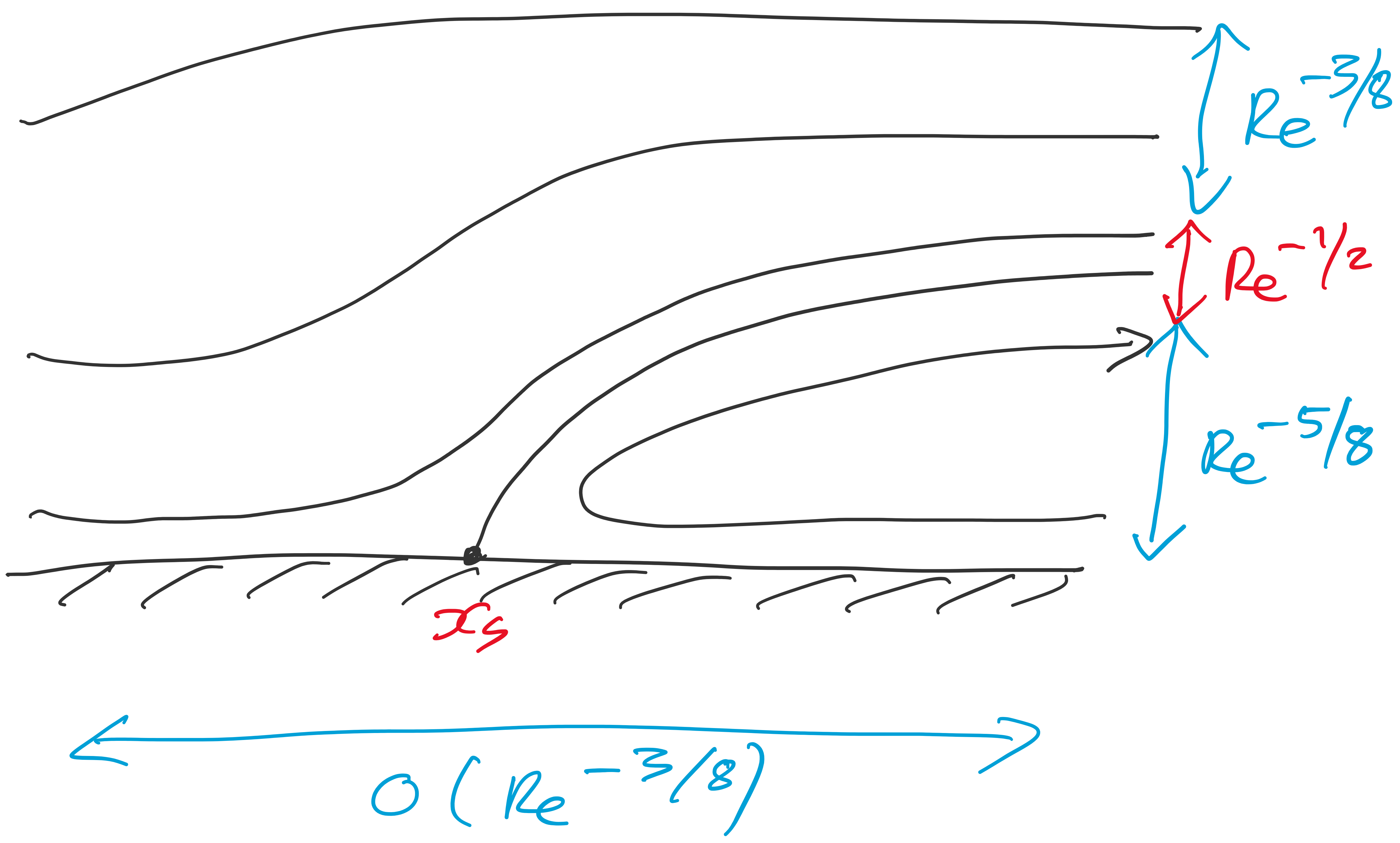


Figure from Prandtl's 1905 paper

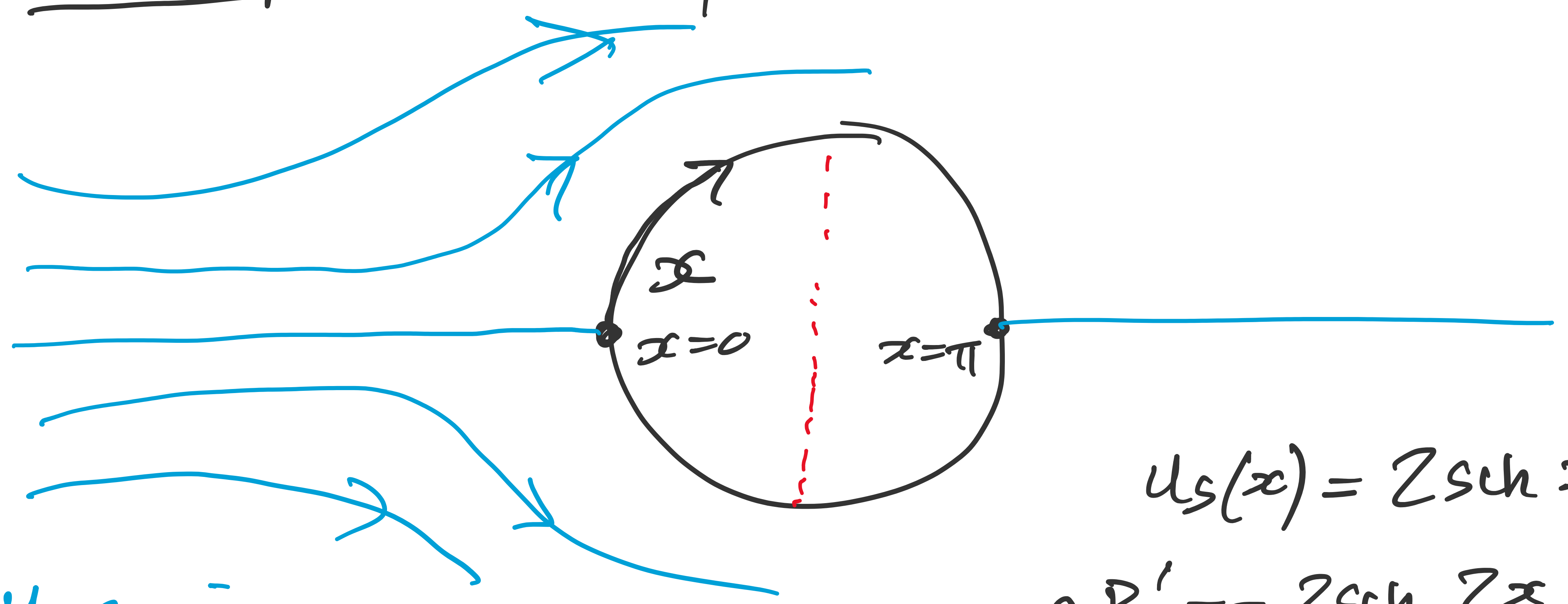
A local analysis near $(x_s, 0)$ shows that we need a more complicated "triple deck" boundary layer structure to describe how the viscous boundary layer separates from the wall (F.T. Smith 1977, K. Stewartson 1981)



Physically, the flow in the boundary layer loses momentum due to viscous wall friction, so it cannot keep up with the fluid in the outer flow pushing against the adverse pressure gradient.

Instead, the boundary layer separates, carrying vorticity from the boundary into the outer flow. This invalidates Prandtl's picture of an outer potential flow.

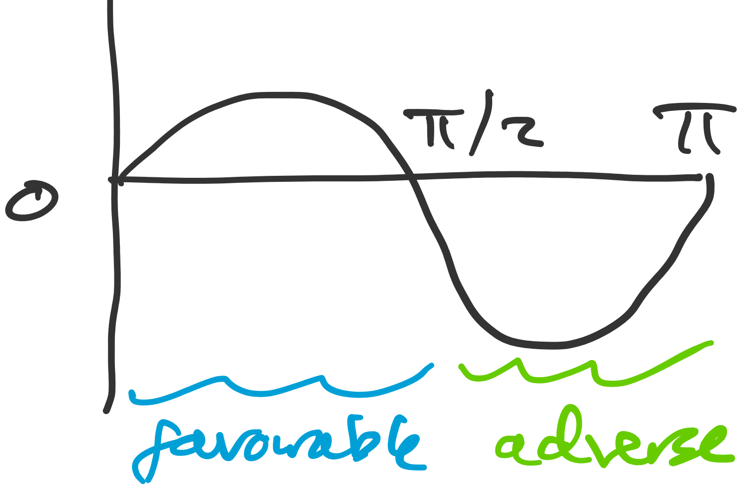
Example: flow past a circular cylinder



$\underline{u} \sim \underline{i}$

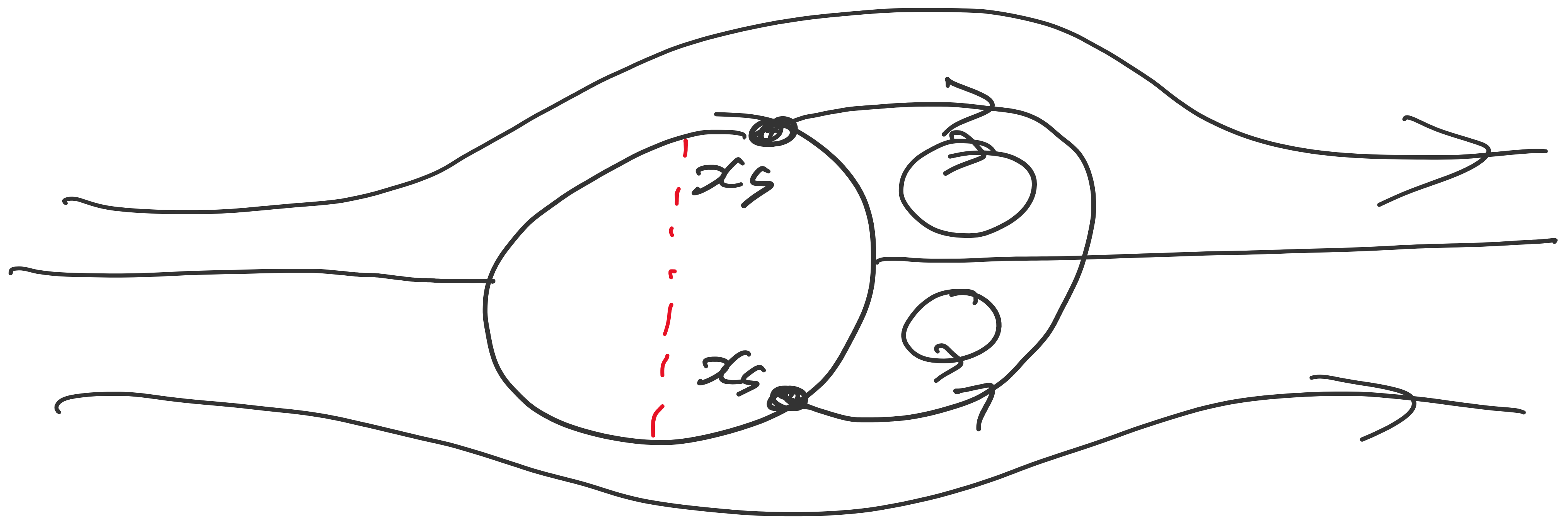
uniform oncoming flow

$u_s(x) = 2\sin x$

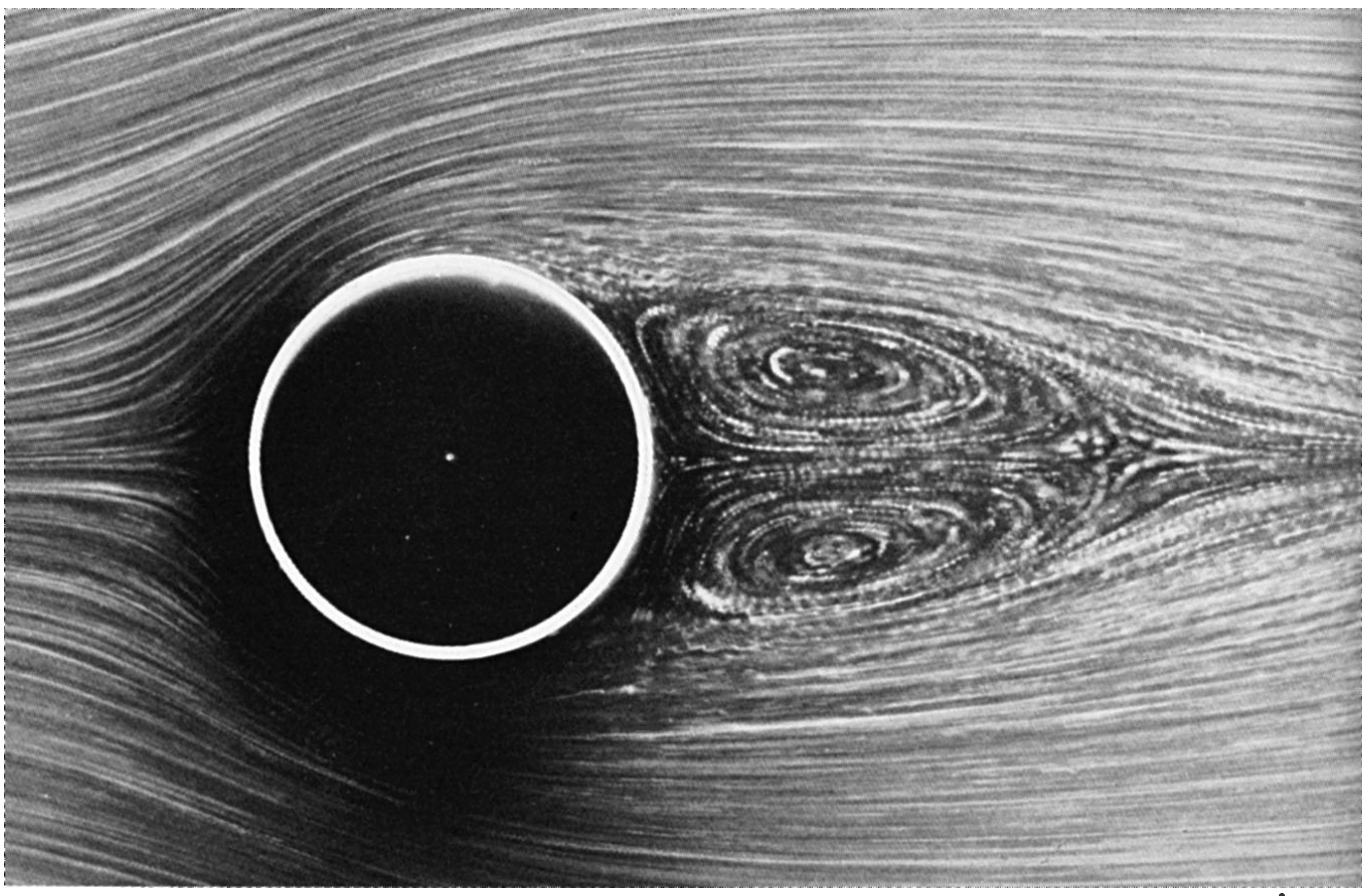


Numerical solutions of the boundary layer equations give: $x_s \simeq 1.815$.

Pretty good approximation for $Re \lesssim 100$.



Flow past a cylinder at $Re = 26$



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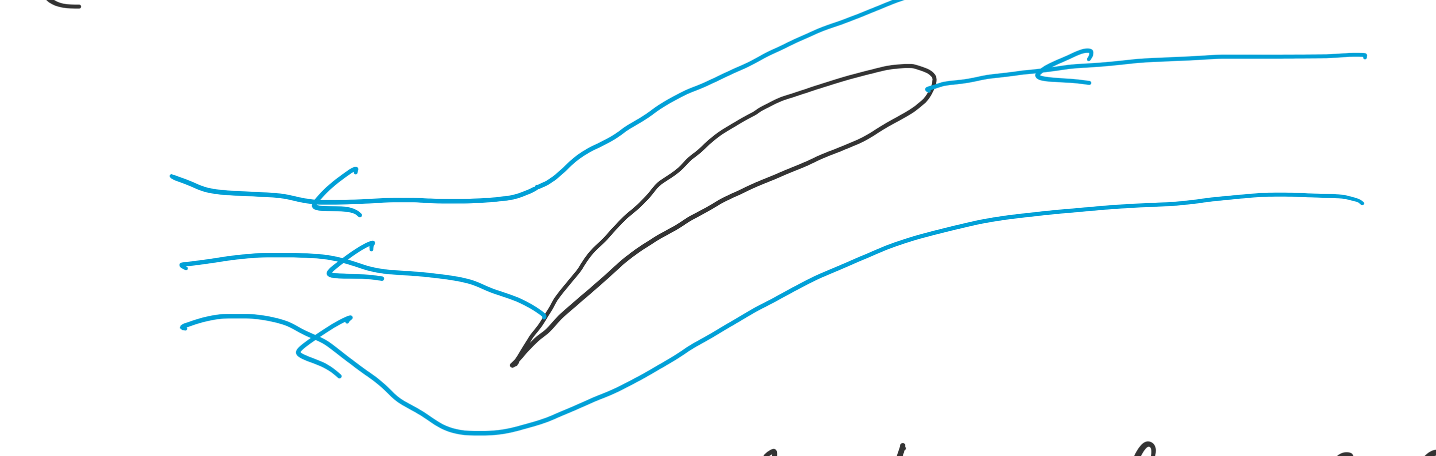
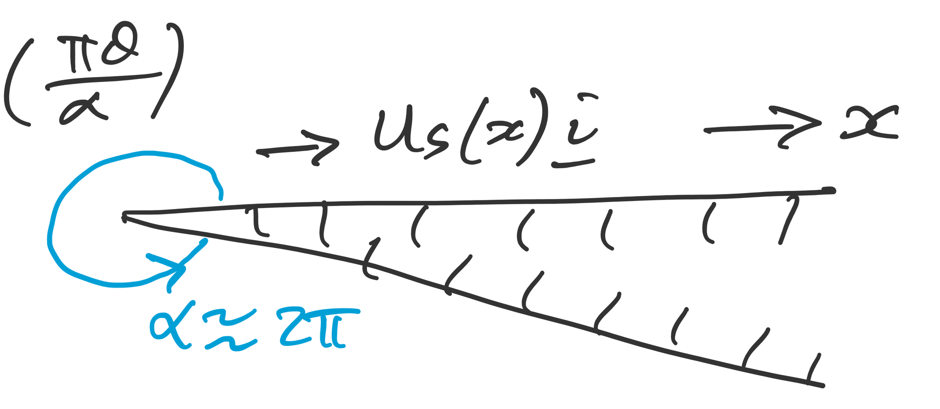
Example: flow past an aerofoil

Inviscid solution with no circulation (as in Part A)



Near the sharp trailing edge we can use potential flow around a (very large, small angle) wedge, and the Falkner–Skan problem for $U_s(x) = x^m$ with $m<0$.

$$\phi(r,\theta) \simeq r^{\pi/\alpha}\cos\left(\frac{\pi\theta}{\alpha}\right)$$



$$U_s(x) = \left.\frac{\partial\phi}{\partial r}\right|_{r=x,\,\theta=0}$$

$$\propto x^m \quad \text{with } m = \frac{\pi}{\alpha} - 1 \simeq -\frac{1}{2}$$

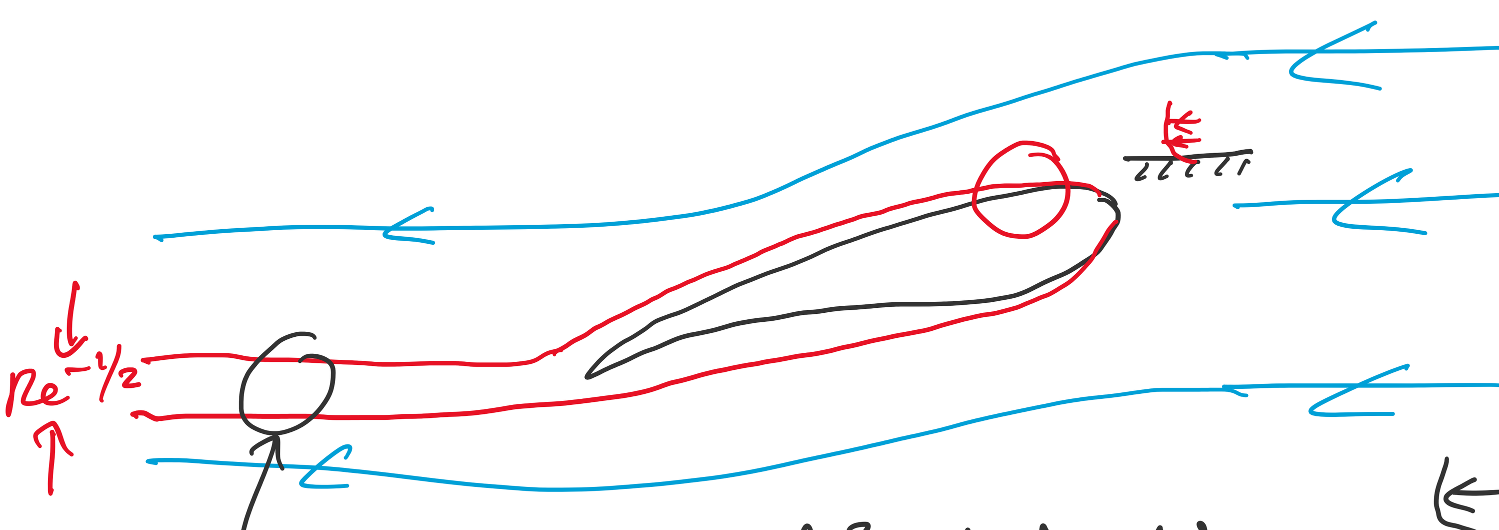
This puts us on the $m<-0.0904$ regime, so there's no solution with an attached boundary layer.

The adverse pressure gradient causes the BL to separate.

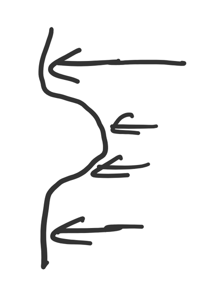
The only way to avoid separation is to impose the Kutta–Joukowski condition:

The circulation Γ around the aerofoil is such that the velocity is finite at the trailing edge.

The boundary layer then detaches smoothly from the trailing edge into a thin viscous wake:



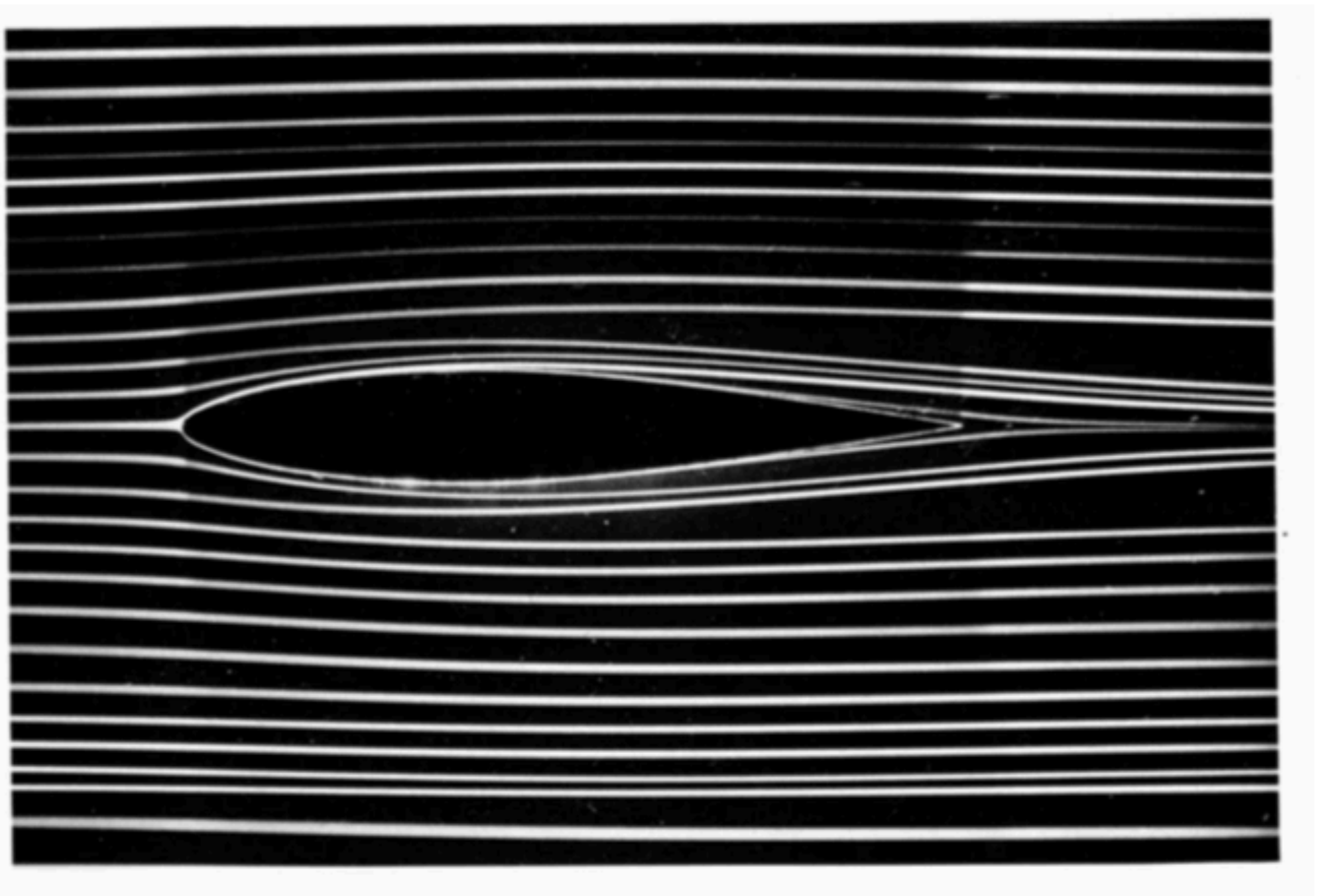
The flow profile looks like:



The flow in the viscous wake is slower, having been slowed by viscous friction with the aerofoil.

This flow structure is observed for blunt nosed aerofoils at moderate Re and shallow angles of attack between the aerofoil and the oncoming flow.

Flow past a horizontal symmetric aerofoil stays attached as in the above theory:



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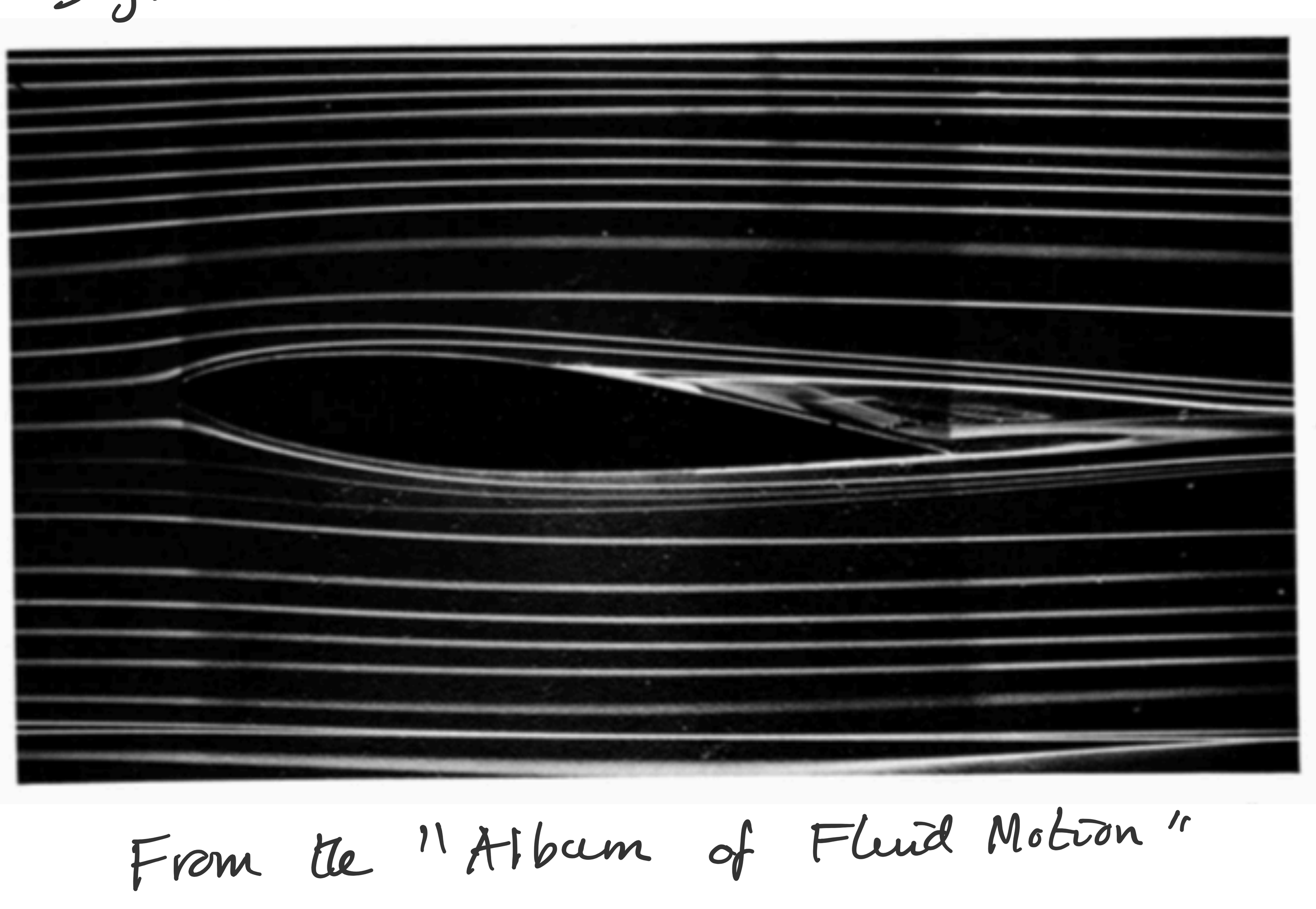
Tilting the aerofoil slightly causes the flow over the top to separate about half-way along. This is why real aerofoils are not up-down symmetric.



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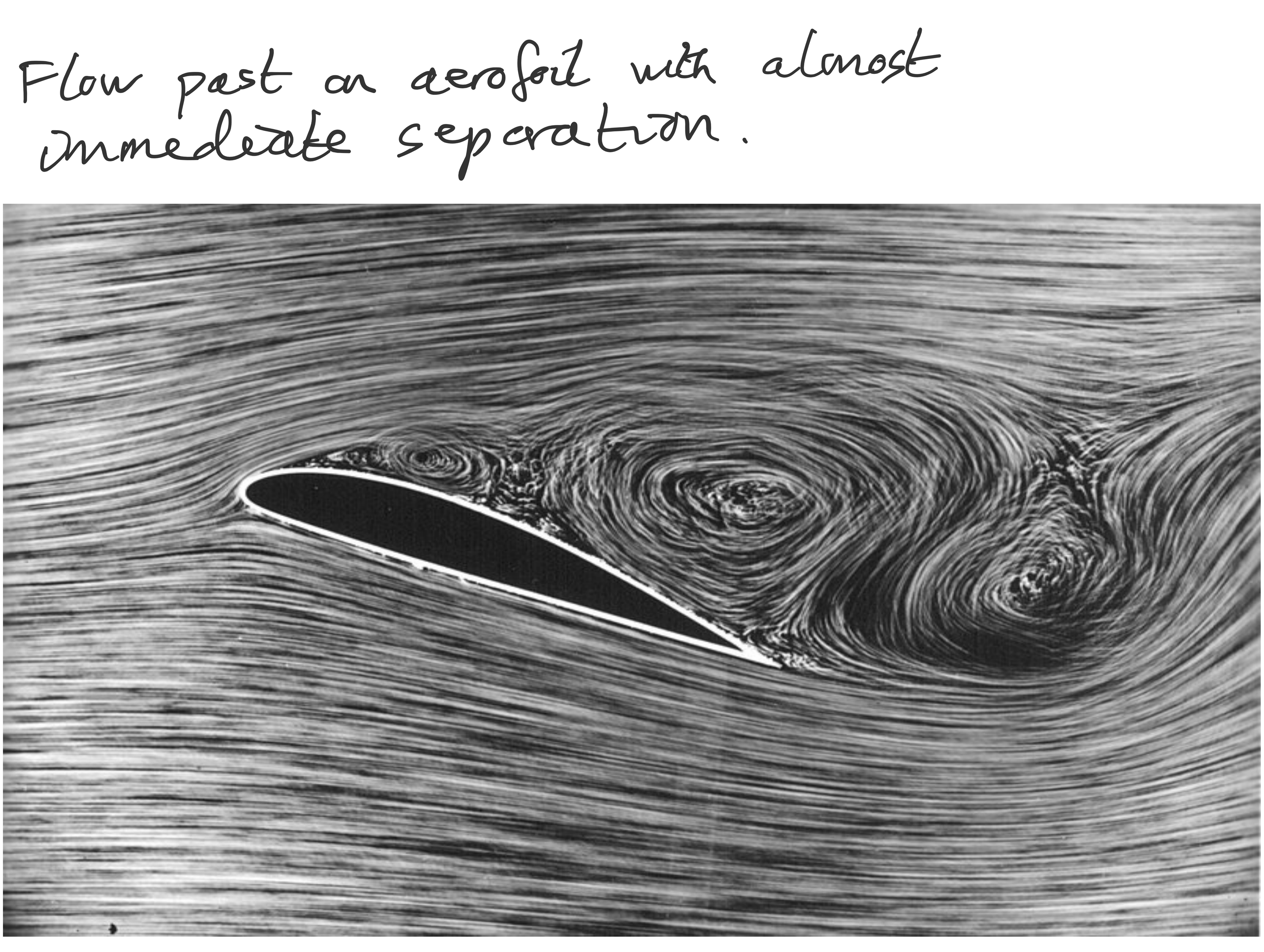
At larger angles of attack (trying to get more lift) the boundary layer separates earlier, forms a large turbulent wake and a sudden increase in drag and loss of lift.

Flow past an aerofoil with almost immediate separation.



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