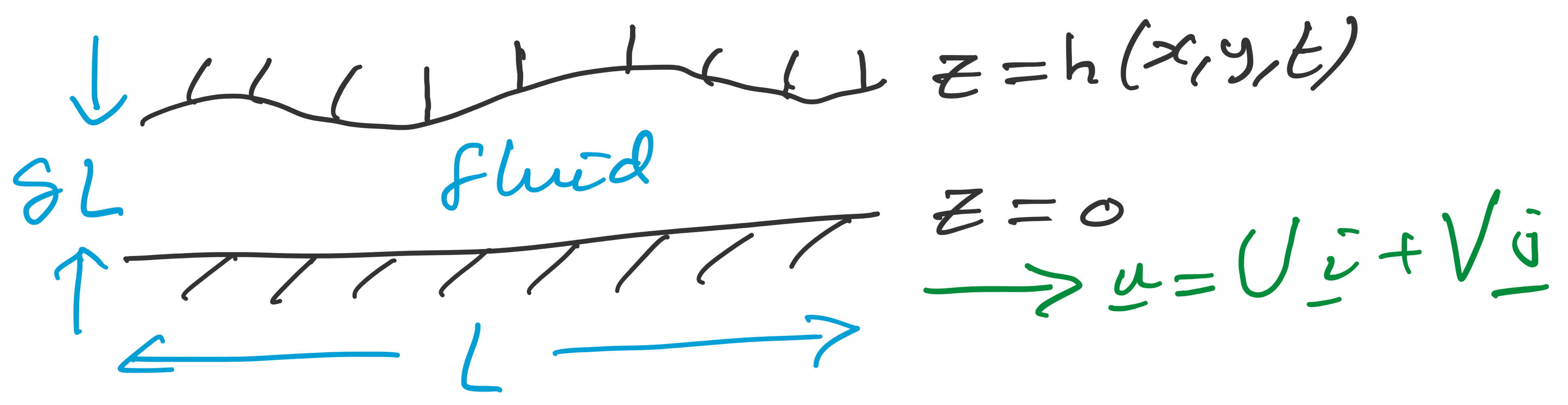
Sunday, 22 November 2020 21:33

Viscous Flow Lecture 14
Lubrication theory examples

Last time: Reynolds' lubrication equation

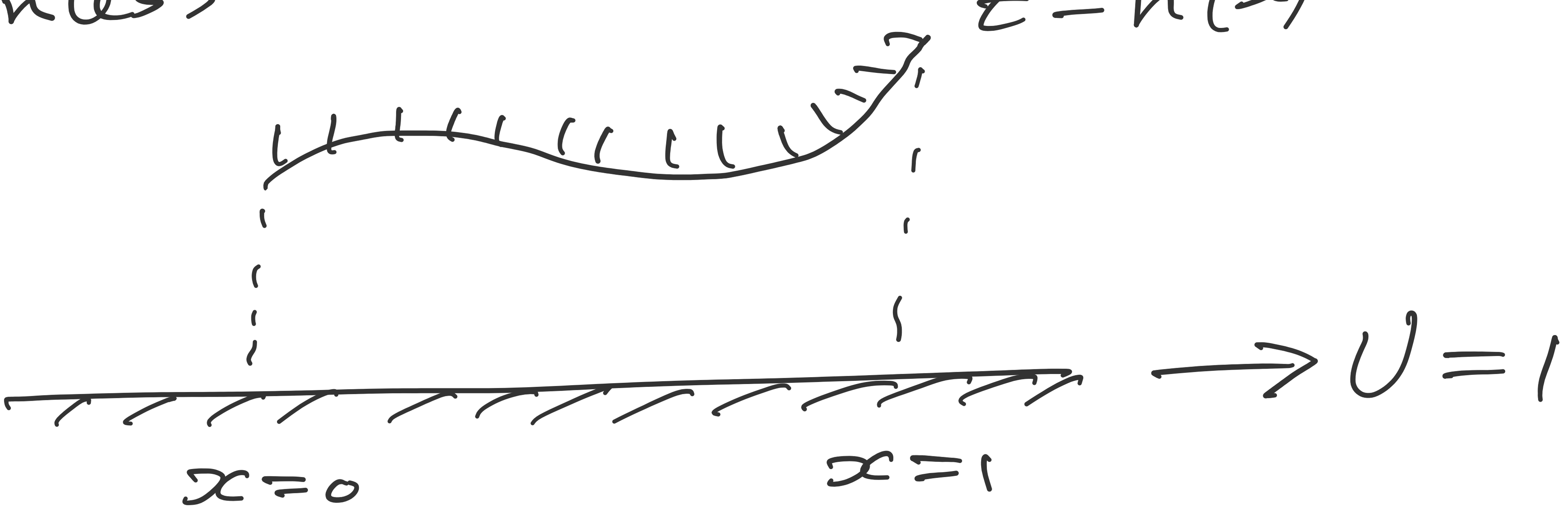
$$\frac{\partial}{\partial x}\left(\frac{h^3}{12}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{h^3}{12}\frac{\partial p}{\partial y}\right)$$
$$=\frac{\partial h}{\partial t}+\frac{U}{2}\frac{\partial h}{\partial x}+\frac{V}{2}\frac{\partial h}{\partial y}$$



Example: steady flow in a 1D slider bearing

Set $h = h(x)$ on $0 < x < 1$
$U = 1$ and $V = 0$

Dimensionless problem



(RLE) becomes
$$\frac{d}{dx}\left(h^3(x)\frac{dp}{dx}\right)=6\frac{dh}{dx}$$

Assume the two ends at $x = 0, 1$ are at ambient atmospheric pressure, $p(0) = p(1) = 0$.

This is a Sturm-Liouville problem for $p(x)$. In fact, we can just integrate w.r.t. x:

integration constant

$$h^3\frac{dp}{dx}=6(h-h_0)$$

$$p(x)=\int_0^x \frac{6(h(s)-h_0)}{h(s)^3}\,ds$$

using the $p(0)=0$ boundary condition to set the lower integration limit.

The arbitrary constant h_0 is fixed by imposing $p(1) = 0$, which gives

$$h_0\int_0^1\frac{ds}{h(s)^3}=\int_0^1\frac{ds}{h(s)^2}$$

Returning to dimensional variables: z-force (load) on the slider is

$$F_z \sim \frac{\mu U}{\delta^2 L}L\int_0^1 p(x)\,dx$$

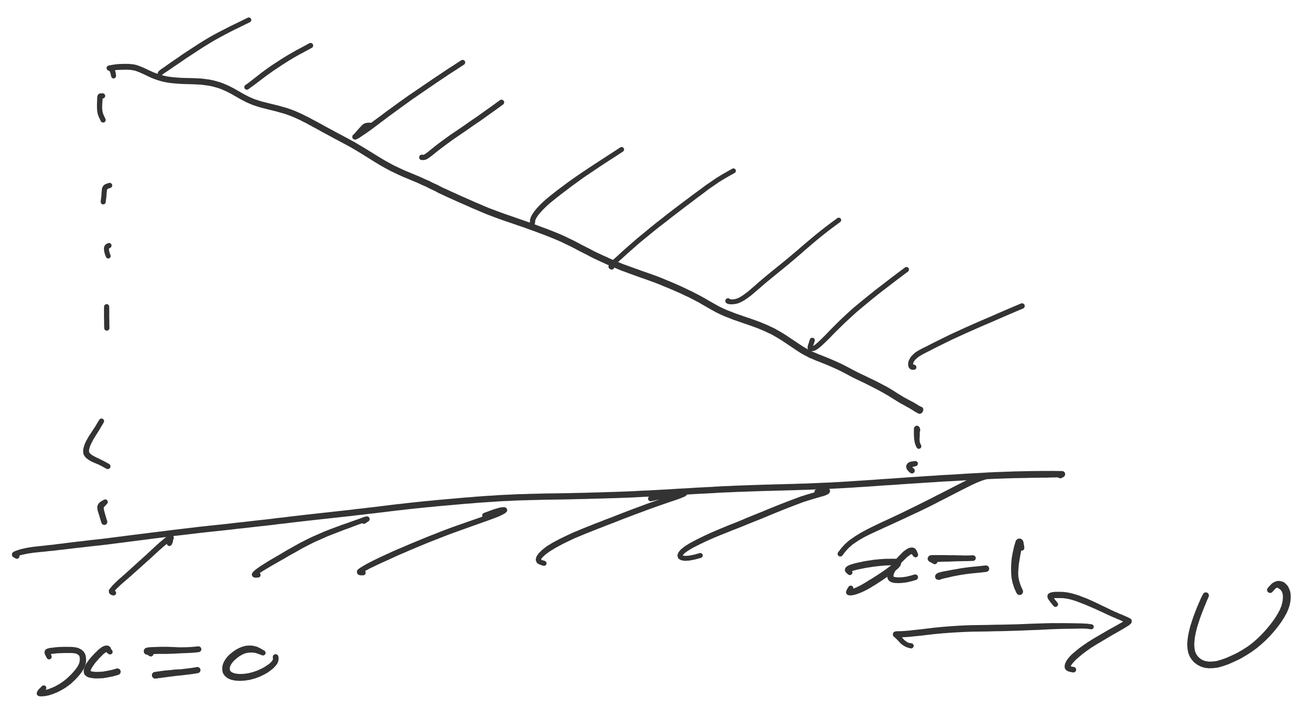
x-force (friction) on the slider is

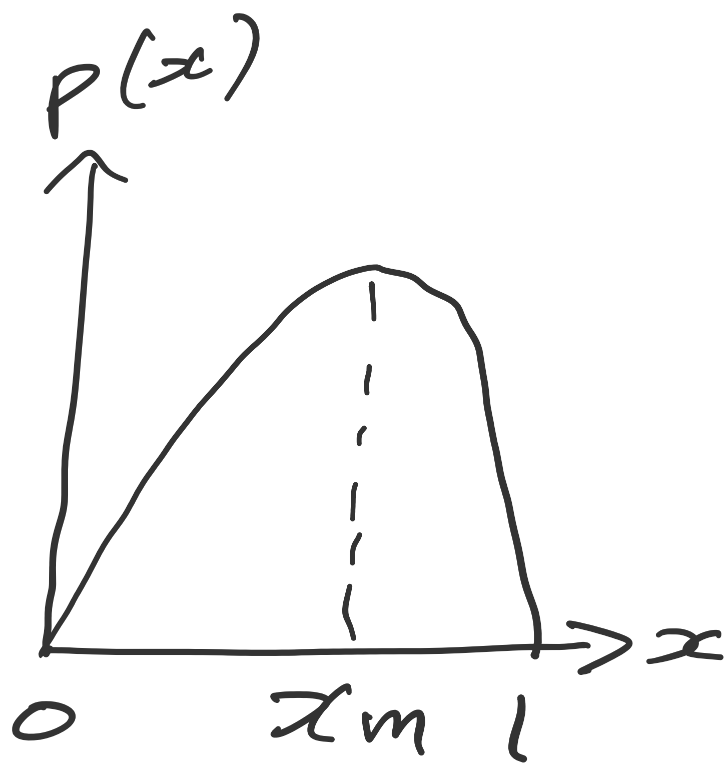
$$F_x \sim -\frac{\mu U}{\delta L}L\int_0^1 p\frac{\partial h}{\partial x}+u_z\Big|_{z=h}\,dx$$

$F_x = O(\delta F_z)$ so load $\gg$ friction

Sunday, 22 November 2020 21:33

Case 1) Converging channel

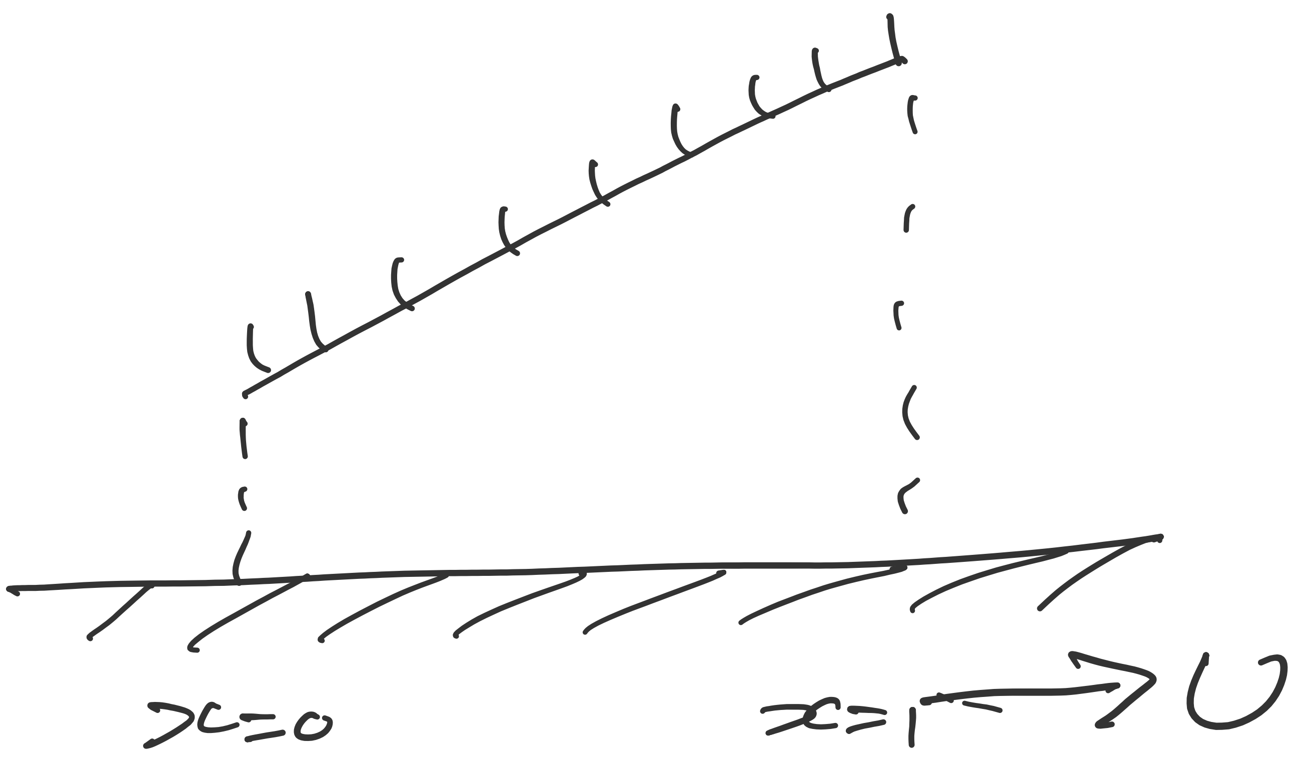


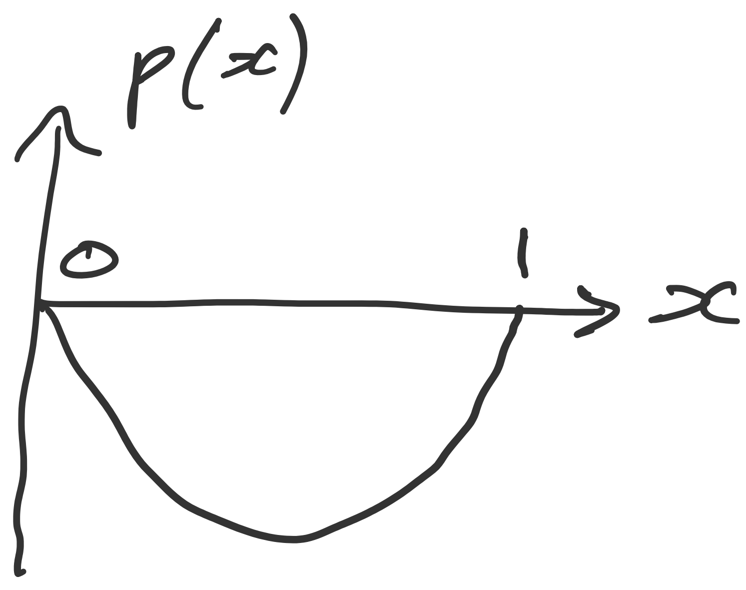


$h(x_m) = h_0$

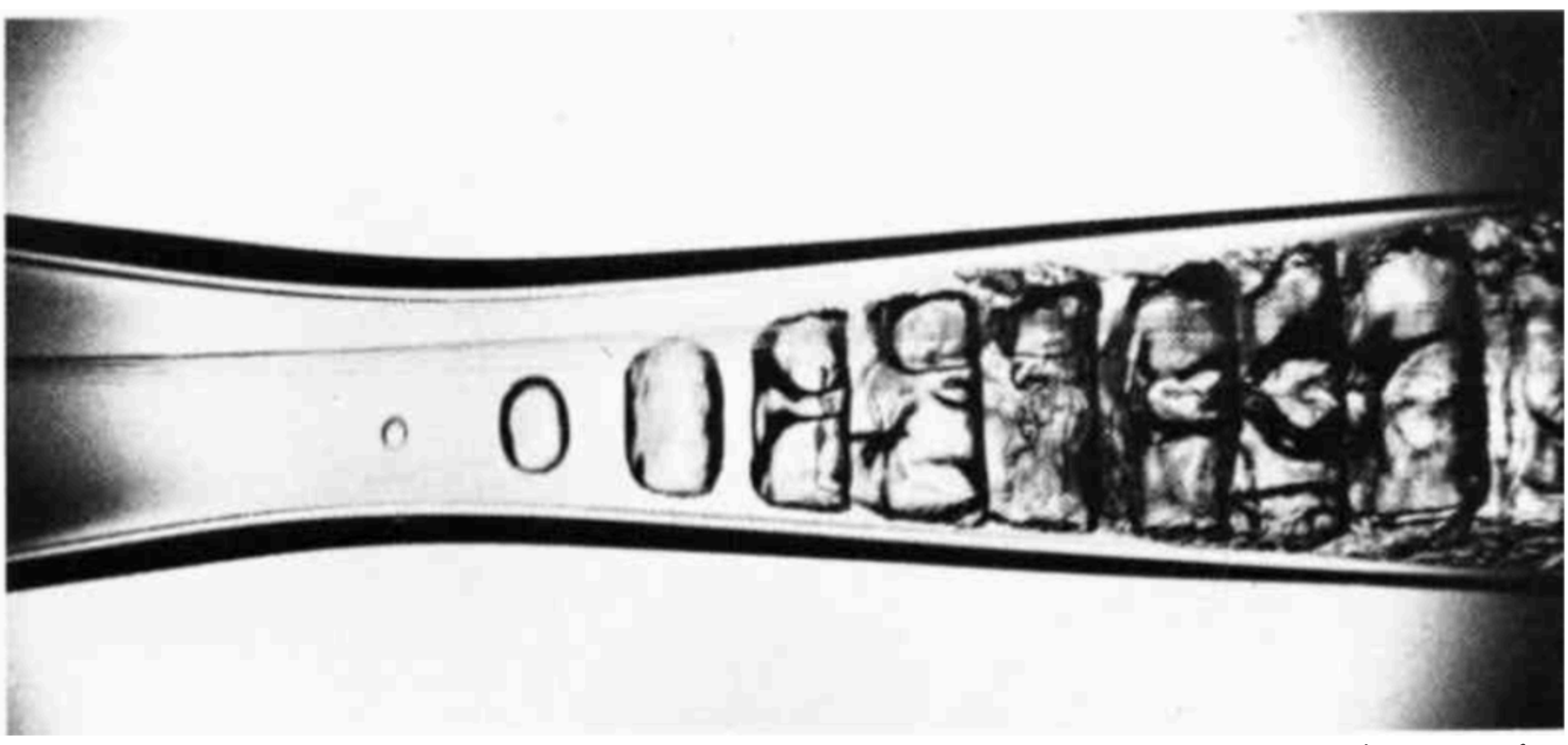
$p > 0$, supports a load

Case 2) Diverging channel



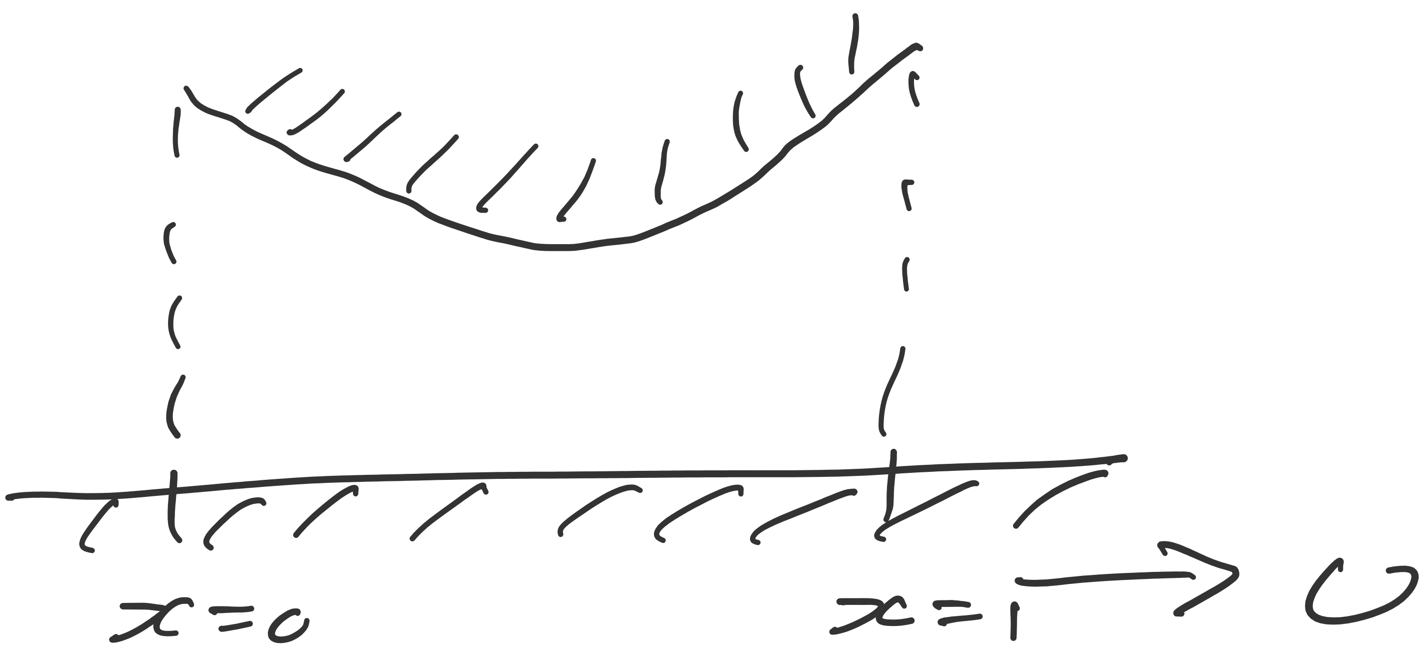


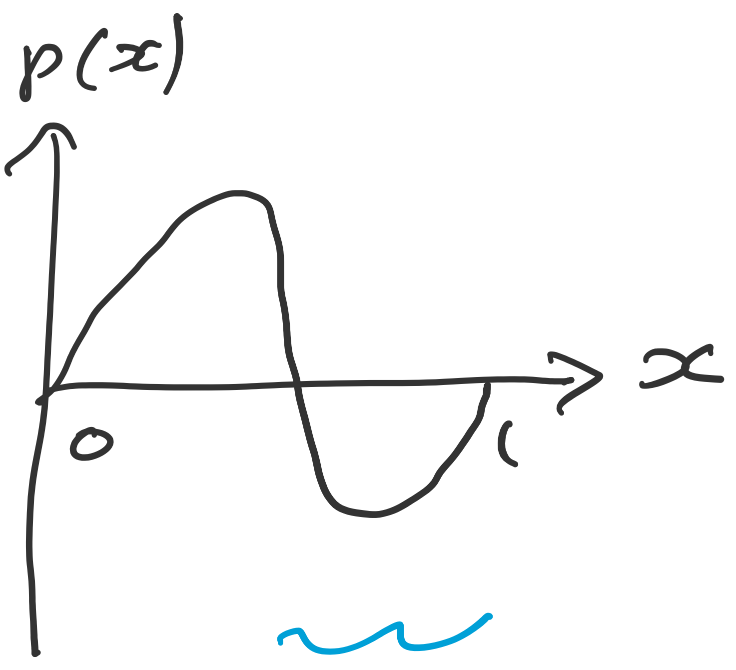
$p < 0$. Fluid may cavitate, i.e. form gas or vapour bubbles, if the dimensional pressure becomes too small (like lowering the boiling point).



From the "Album of Fluid Motion"

Case 3) A non-monotone channel

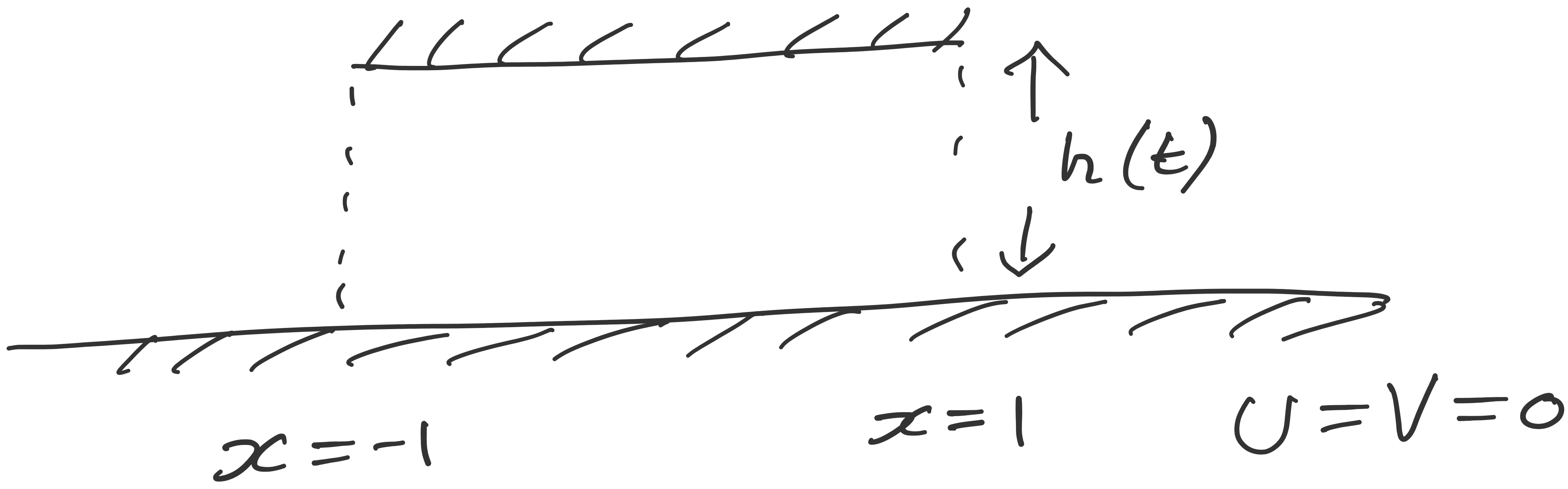




may cavitate here

Load is supported if
$$\int_0^l p(x)\,dx > 0$$
but the fluid may cavitate, invalidating the incompressibility assumption.

Example: A 1D squeeze film



(RLE) with $p = p(x,t)$, $h = h(t)$ gives

$$\frac{\partial}{\partial x}\left(\frac{h^3}{12}\frac{\partial p}{\partial x}\right) = \frac{dh}{dt} = \dot{h}.$$

Assume ambient pressure at the two ends, so $p(\pm 1) = 0$.

$$\frac{\partial^2 p}{\partial x^2} = \frac{12\dot{h}}{h^3}$$

$$\Rightarrow \quad p(x,t) = -\frac{6\dot{h}}{h^3}(1 - x^2)$$

The dimensionless load is

$$F_z = \int_{-1}^{1} p\, dx = -\frac{8\dot{h}}{h^3} = \frac{d}{dt}\left(\frac{4}{h^2}\right)$$

The pressure and the load both have the opposite sign to $\dot{h}$. Both are positive when the gap is narrowing (i.e. $\dot{h} < 0$).

Imposing a constant force F_z, say a weight on the upper boundary, gives

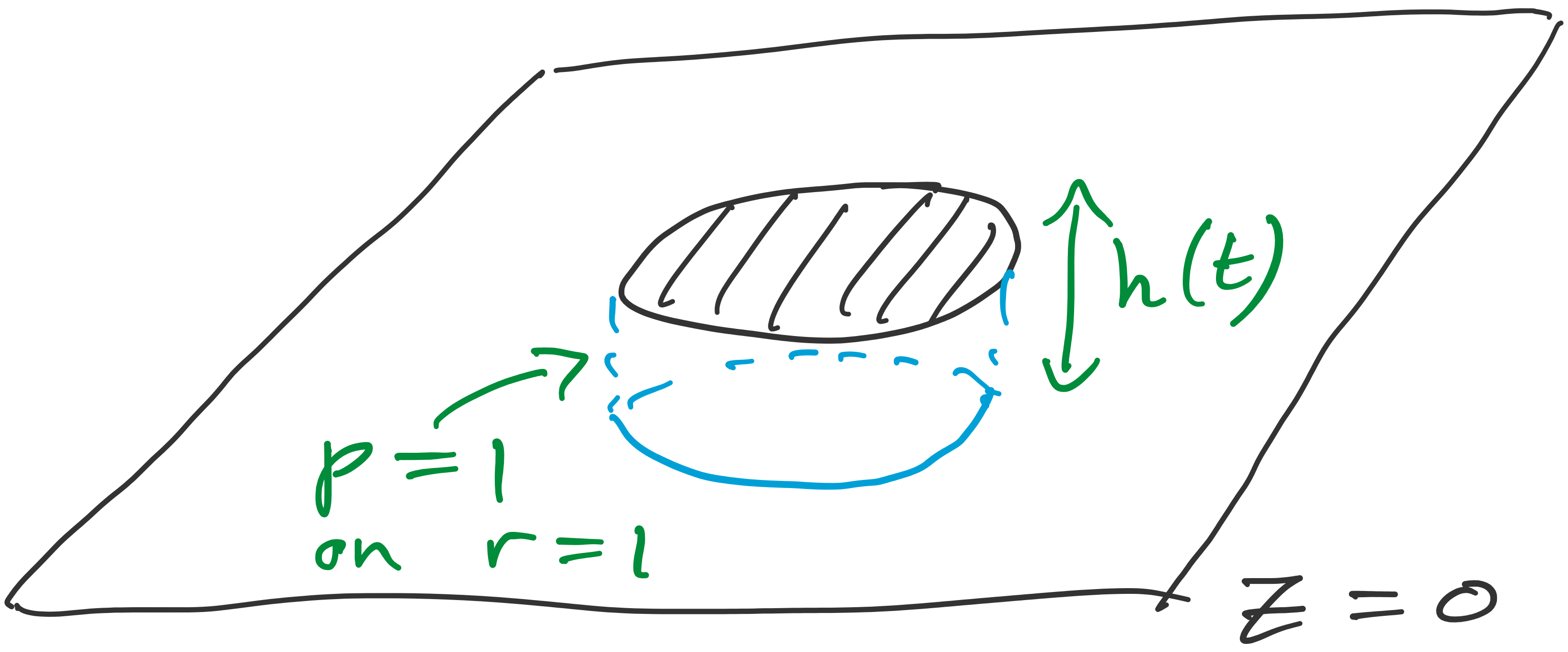
$$F_z\, t = \frac{4}{h^2} - \frac{4}{h(0)^2}$$

height h at $t = 0$

$h \to 0$ as $t \to \infty$, but it takes an infinitely long time to squeeze out all the fluid.

Sunday, 22 November 2020 21:33

Example: Axisymmetric squeeze film



$$(RLE) \Rightarrow \nabla \cdot \left(\frac{h^3}{12} \nabla p \right) = \dot{h}$$

The axisymmetric version for $p(r,t)$ gives

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{h^3}{12} \frac{\partial p}{\partial r} \right) = \dot{h}$$

in $0 \le r \le 1$

$$\frac{\partial}{\partial r} \left(r \frac{\partial p}{\partial r} \right) = \frac{12 \dot{h}}{h^3} r$$

$$p = \frac{3 \dot{h}}{h^3} \left(r^2 + A(t) \log r + B(t) \right)$$

We need p to be bounded as $r \to 0$, so $A(t) = 0$, and $p = 0$ on $r = 1$, which determines $B(t) = -1$,

so $$p = -\frac{3\dot{h}}{h^3}(1 - r^2)$$

The load is

$$F_z = \int_0^{2\pi} d\theta \int_0^1 dr \, r p$$

$$= -\frac{3\pi}{2} \frac{\dot{h}}{h^3}$$