

Problem Sheet 1

Please, if your class is in week 2, read the slides of Lectures 1 and 2 before attempting this sheet.

(1) (a) Which of the following are formulas of $\mathcal{L}$? Give reasons.

- (i) $(p_3 \rightarrow p_1)$
- (ii) $p_1 \rightarrow p_2 \rightarrow p_3$
- (iii) $(\neg p_5 \wedge \neg p_6) = \neg p_{11}$
- (iv) $(p \leftrightarrow \neg p)$
- (v) $((p_1 \vee \neg p_1) \rightarrow (\neg p_2))$

(b) Prove carefully that for any formula ϕ , the number of left parentheses occurring in ϕ is equal to the number of right parentheses occurring in ϕ .

(2) (a) Prove that the length of a formula with exactly n occurrences of the negation symbol and m occurrences of binary connectives is $4m + n + 1$. Check this for the formulas in question (1) (a).

(b) List all formulas of $\mathcal{L}$ of length ≤ 6 .

(3) Can a proper initial segment of a formula ever be a formula again? How about final proper segments?

(4) Prove the Unique Readability Theorem.